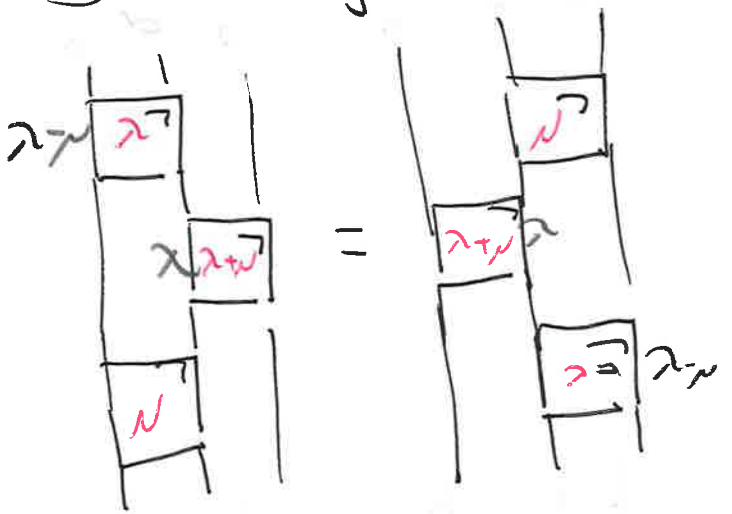
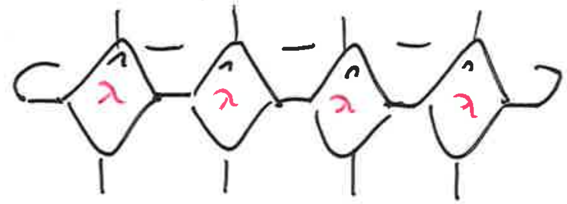
• Braiding



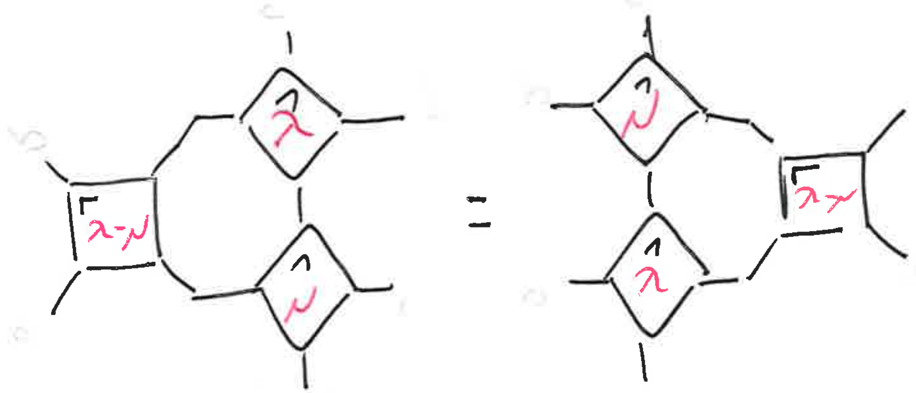
$\left\{\begin{array}{l}\text{cons charges (local)}\\ \text{eigenstates / Bethe states}\end{array}\right.$

⇝ transfer matrix

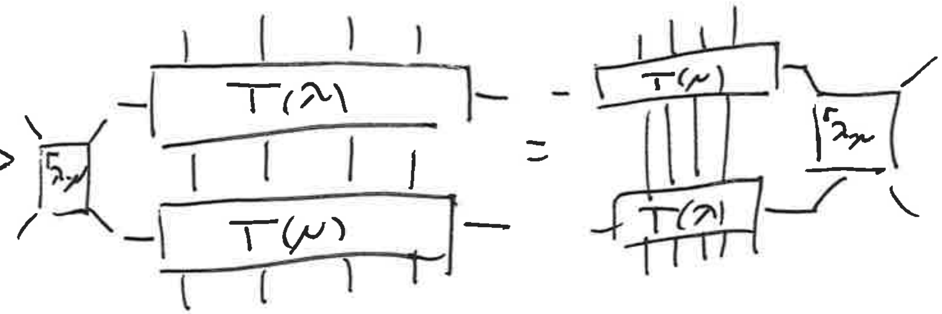


$= \tau(\lambda)$

⇝ $[\tau(\lambda), \tau(\mu)] = 0 \quad \forall \lambda, \mu$



⇝



⇝

* Local hamiltonians?

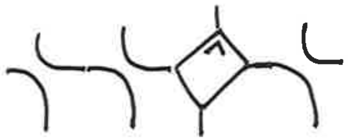
Choose $R(\lambda=0) = \mathbb{1}$

$$Q_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \ln[\tau(\lambda)]$$

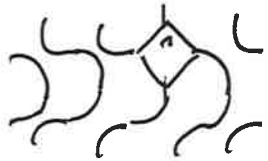
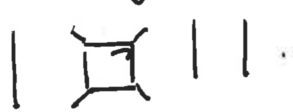
$\rightsquigarrow \tau(0) =$  $=$ Translation

$\rightsquigarrow Q_1 = \tau^{-1}(0)\, \partial_\lambda \tau|_{\lambda=0}$

n^{th} site

$\rightsquigarrow \partial_\lambda \tau|_{\lambda=0} = \sum_n$  $\in$

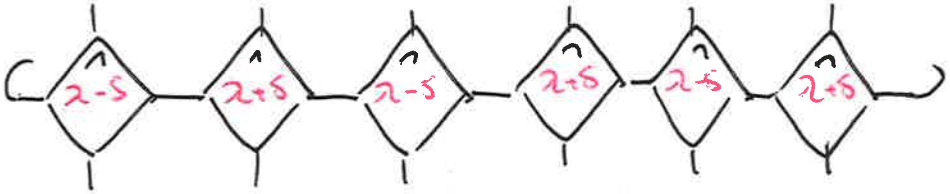
n^{th} site

$\rightsquigarrow \tau^{-1}(0)\, \partial_\lambda \tau|_{\lambda=0} = \sum_n$  $= \sum_n$  $= \sum_n h_{n,n+1}$

Higher-order terms also local

* Circuit formalism

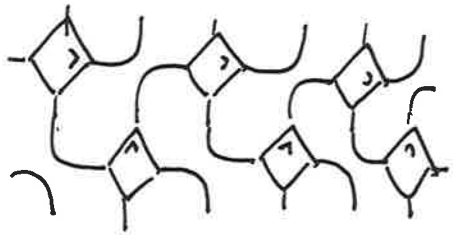
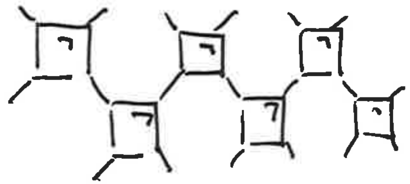
⇝ inhomogeneities don't change commutation

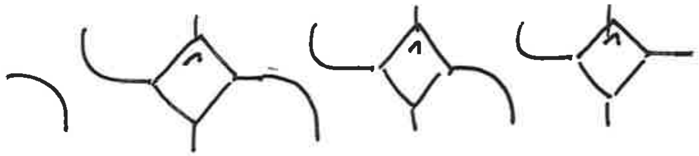
$\tau(\lambda) =$ 

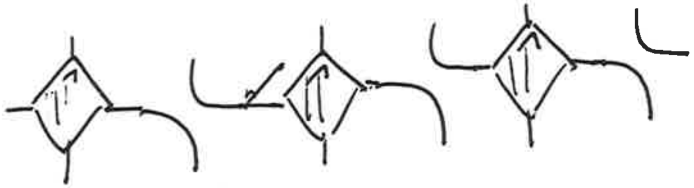
$R(0) = \mathbb{1}$

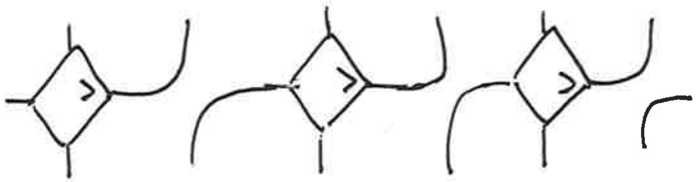
$R(2\delta) = U$

$R(-2\delta) = U^+$

$\rightsquigarrow \tau^{-1}(-\delta)\,\tau(\delta) =$  $=$ 

$\tau(\delta) =$ 

$\tau(-\delta) =$  $\rightsquigarrow \tau^{-1}(-\delta)\,\tau(\delta)$

$\tau^{-1}(-\delta) =$ 

$$R(\lambda) = \frac{\mathbb{1} + i\lambda S}{1 + i\lambda}$$

$$R(-\lambda) = R(\lambda)^{+}$$

$$R(\lambda)R^{+}(\lambda) = \frac{1}{1+\lambda^2}\left(\mathbb{1} + i\lambda S\right)\left(\mathbb{1} - i\lambda S\right) = \frac{1}{1+\lambda^2}\left[1 + i\lambda S - i\lambda S + \lambda^2 \mathbb{1}\right] = \mathbb{1}$$

PKX

XXZ

$$R(\lambda) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin(\gamma)/\sin(\gamma+\lambda) & \sin(\lambda)/\sin(\gamma+\lambda) & 0 \\ 0 & \sin(\lambda)/\sin(\gamma+\lambda) & \sin(\gamma)/\sin(\gamma+\lambda) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

γ fixed parameter, related to Δ

$\rightsquigarrow R(\lambda=0) = \mathbb{1}$

$\rightsquigarrow$ unitary at $\lambda = iu$, $u \in \mathbb{R}$

$$\sin\gamma \rightsquigarrow i \sinh u$$
$$\sin(\gamma+\lambda) = \sin\gamma \cosh u + i \cos\gamma \sinh u$$

$\rightsquigarrow$ Returns XXZ gate for $\exp[i\tau(XX+YY+\Delta ZZ)]$

$$\gamma = \arccos\left[\sin(\Delta\tau/2)/\sin(\tau/2)\right] \sim \arccos(\Delta) \text{ for } \tau \to 0$$
$$u = \operatorname{arcsinh}\left[\sin(\gamma)\tan(\tau/2)\right]$$

$$\sigma^x\sigma^x + \sigma^y\sigma^y + \Delta(\sigma^z\sigma^z - 1\!\!1)$$

$$= 2\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\Delta & 1 & 0 \\ 0 & 1 & -\Delta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

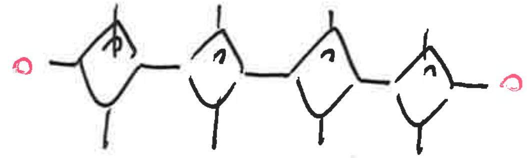
$$\partial_\lambda R\big|_{\lambda=0} = \frac{1}{\sin\gamma}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\cos\gamma & 1 & 0 \\ 0 & 1 & -\cos\gamma & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rightsquigarrow \Delta = \cos(\gamma)$$

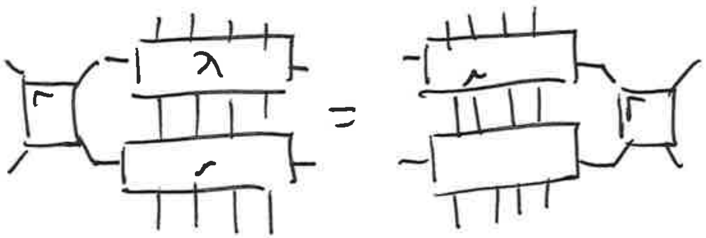
- Algebraic Bethe ansatz

$\rightsquigarrow$ use algebra spanned by elements of $T(\lambda)$ [= monodromy matrix]

$$T(\lambda) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}$$


$= A(\lambda)$

$$\tau(\lambda) = A(\lambda) + D(\lambda)$$



$$R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b(\lambda,\mu) & c(\lambda,\mu) & 0 \\ 0 & c(\lambda,\mu) & b(\lambda,\mu) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$b = \frac{\lambda-\mu}{\lambda-\mu+\eta}; \quad c = \frac{\eta}{\lambda-\mu+\eta}$$

$$\rightsquigarrow A(\lambda)B(\mu) = \frac{1}{b(\mu,\lambda)} B(\mu)A(\lambda) + - \frac{c(\mu,\lambda)}{b(\mu,\lambda)} B(\lambda)A(\mu)$$

$$\rightsquigarrow D(\lambda)B(\mu) = \frac{1}{b(\lambda,\mu)} B(\mu)D(\lambda) - \frac{c(\lambda,\mu)}{b(\lambda,\mu)} B(\lambda)D(\mu)$$

$$\rightsquigarrow B(\mu)B(\lambda) = B(\lambda)B(\mu)$$

$\rightsquigarrow$ vacuum state

$A(\omega)|0\rangle = a(\omega)|0\rangle$; $D(\omega)|0\rangle = d(\omega)|0\rangle$; $C(\omega)|0\rangle = 0$; $B(\omega)|0\rangle \neq 0$

$\rightsquigarrow$ Bethe states

$$|\lambda_1 \dots \lambda_N\rangle = \prod_{j=1}^{N} B(\lambda_j)|0\rangle$$

$$\rightsquigarrow A(\lambda)\prod_{j=1}^{N} B(\lambda_j)|0\rangle = \left[a(\lambda)\prod_{k=1}^{N}\frac{1}{b(\lambda_k,\lambda)}\right]\prod_{j=1}^{N} B(\lambda_j)|0\rangle$$
$$+ \; - \sum_{\ell=1}^{N}\left[a(\lambda_\ell)\frac{c(\lambda_\ell,\lambda)}{b(\lambda_\ell,\lambda)}\prod_{k\neq\ell}\frac{1}{b(\lambda_k,\lambda_\ell)}\right] B(\lambda)\prod_{j\neq\ell} B(\lambda_j)|0\rangle$$

$$D(\lambda)\prod_{j=1}^{N} B(\lambda_j)|0\rangle = \left[d(\lambda)\prod_{k=1}^{N}\frac{1}{b(\lambda,\lambda_k)}\right]\prod_{j=1}^{N} B(\lambda_j)|0\rangle$$
$$- \sum_{\ell=1}^{N}\left[d(\lambda_\ell)\frac{c(\lambda,\lambda_\ell)}{b(\lambda,\lambda_\ell)}\prod_{k\neq\ell}\frac{1}{b(\lambda_\ell,\lambda_k)}\right] B(\lambda)\prod_{j\neq\ell} B(\lambda_j)|0\rangle$$

$$\rightsquigarrow [A(\lambda)+D(\lambda)]\prod B(\lambda_j)|0\rangle = [\;\dots\;]\prod_{j=1}^{N} B(\lambda_j)|0\rangle$$

provided $\dfrac{a(\lambda_\ell)}{d(\lambda_\ell)}\prod_{k\neq\ell}\dfrac{b(\lambda_\ell,\lambda_k)}{b(\lambda_k,\lambda_\ell)} = -\dfrac{c(\lambda,\lambda_\ell)}{b(\lambda,\lambda_\ell)}\dfrac{b(\lambda_\ell,\lambda)}{c(\lambda_\ell,\lambda)} \iff \dfrac{a(\lambda_\ell)}{d(\lambda_\ell)}\prod_{k\neq\ell}\dfrac{b(\lambda_\ell,\lambda_k)}{b(\lambda_k,\lambda_\ell)} = 1$