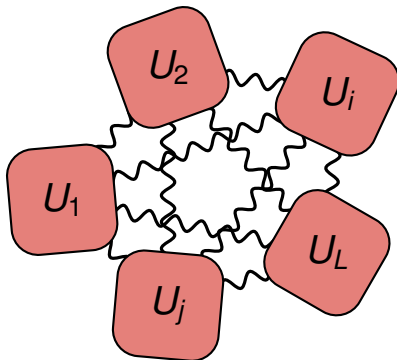




# Universal Spectral Correlations in Interacting Chaotic Quantum Systems

F. Fritzsche, M. F. I. Kieler

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## **Chaotic and Ergodic**

Random Matrix Theory,  
Complexity Growth,  
Scrambling, ...

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Many-Body-Localization,  
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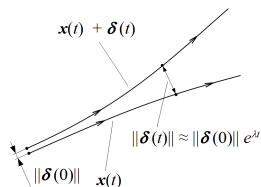
## **Broken Ergodicity**

Many-Body-Localization,  
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Scars, ...

Expectation: Most quantum systems are chaotic.

# How to Define Quantum Chaos?

**Classical chaos** = Sensitive dependence on initial conditions  
= Positive Lyapunov exponents

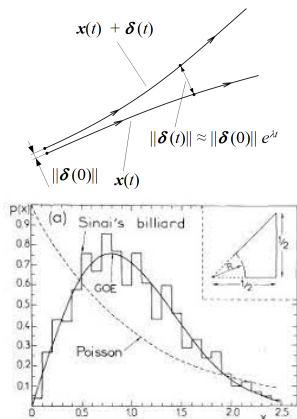


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**Quantum Chaos Conjecture:**  
Spectral statistics follows  
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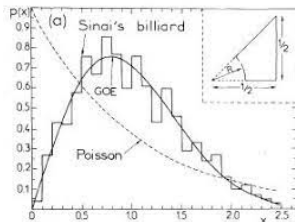
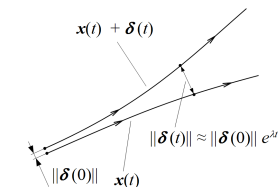
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**Semiclassical periodic orbit theory:**

$$\rho(E) = \bar{\rho}(E) + \sum_{\text{p.o. } \gamma} A_{\gamma} e^{iS_{\gamma}/\hbar}$$



[Bohigas, Giannoni, Schmit, PRL (1984)]

⇒ Correlations in the spectrum  $\sim \rho(E, E')$  agrees with RMT

[Berry, Proc. R. Soc. London (1985)] [Sieber, Richter, Phys. Sc. (2001)] [Müller, et al., PRL (2004)]

## Spectral Form Factor for CUE( $N$ )

- ▶ Floquet systems  $U$  w/o time reversal invariance
- ▶ Correlations between quasi-energies  $\phi_i$ ;  $U|i\rangle = e^{i\phi_i}|i\rangle$ :

$$\rho(\phi, \phi') \xrightarrow{\omega=\phi'-\phi} r_2(\omega) = \left\langle \frac{2\pi}{N^2} \sum_{ij} \delta(\phi_i - \phi_j - \omega) \right\rangle_{\text{CUE}(N)}^{\text{connected}}$$

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- ▶ SFF:  $K_N(t) = \mathcal{F}(r_2)(t) \rightarrow$  correlations at energy scale  $\frac{2\pi}{t}$

$$K_N(t) = \left\langle |\text{tr}(U^t)|^2 \right\rangle_{\text{CUE}(N)} - N^2 \delta_{t0} = \min\{t, N\}$$

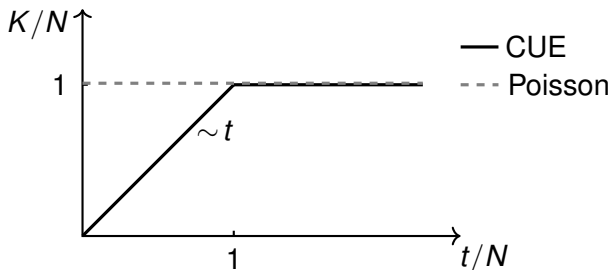
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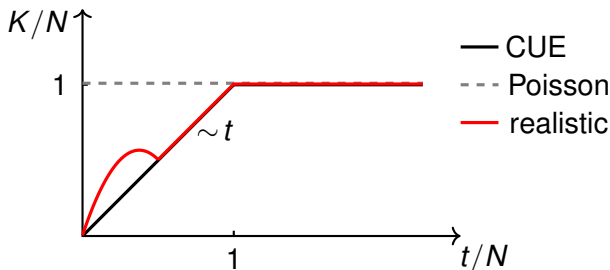
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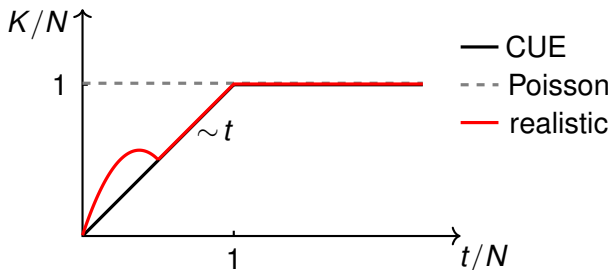
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**Quantum Chaos = Linear Ramp\* of SFF**

\*for  $t >$  non-universal Thouless time



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Typical many-body quantum systems  $\neq$  full random matrices

- ▶ Individual interacting particles/degrees of freedom  
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No universal mechanism, but exact results for specific systems:

- ▶ Dual-unitary circuits with  $L \rightarrow \infty$  [Bertini, Kos, Prosen, Commun. Math. Phys (2021)]
- ▶ Random circuits with  $N \rightarrow \infty$  [Chan, De Luca, Chalker, PRL (2018)]
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**Here: Interacting chaotic systems**

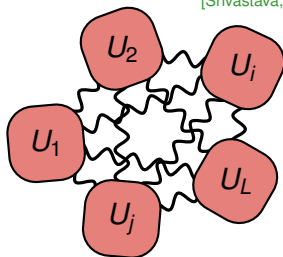
large but finite  $N$ , arbitrary  $L$ , without spatial structure

# Extended Random Matrix Transition Ensemble

[Srivastava, *et al.*, PRL, 2016]

Floquet systems:

$$U = U_c(\epsilon) (U_1 \otimes U_2 \otimes \cdots \otimes U_L)$$



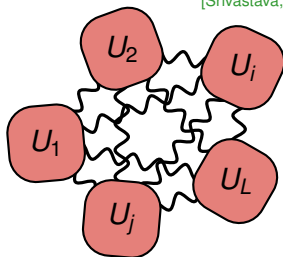
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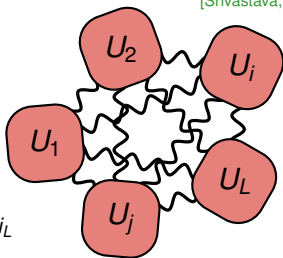
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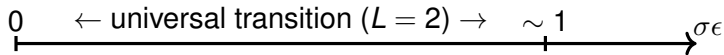
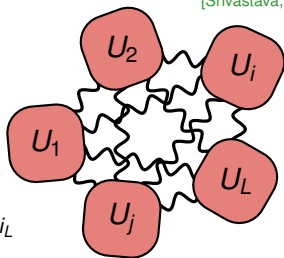
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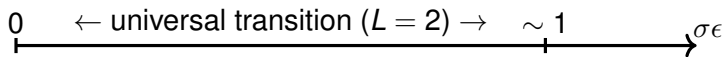
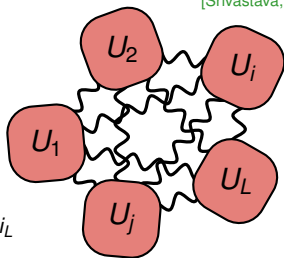
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CUE( $N^2$ ) spectrum

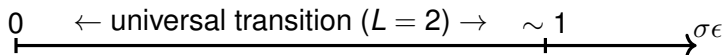
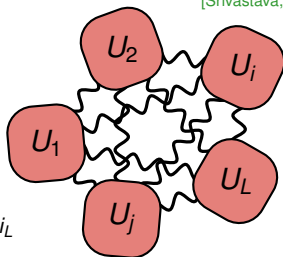
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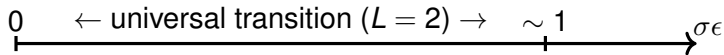
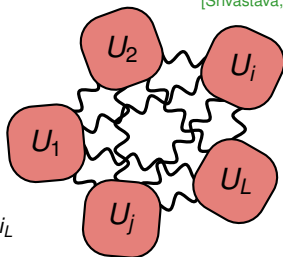
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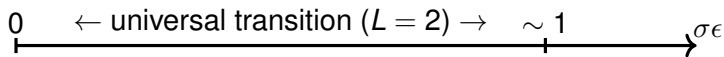
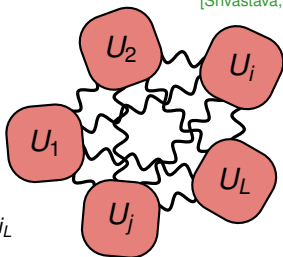
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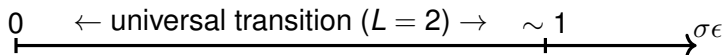
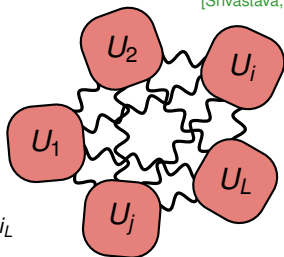
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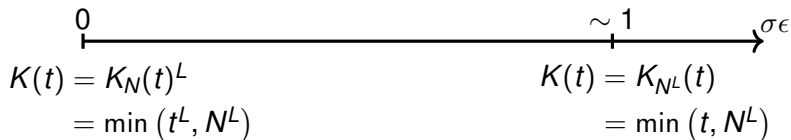
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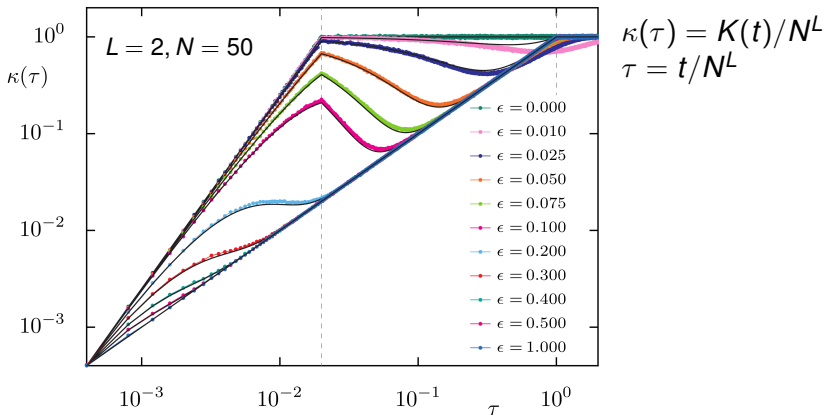
**Here: Universal transition of the SFF for any  $L$**

## Spectral Form Factor for RMTE



# Spectral Form Factor for RMTE

$$\begin{array}{ll} 0 & \sim 1 \\ \left| \right. & \left. \right| \sigma \epsilon \\ K(t) = K_N(t)^L & K(t) = K_{N^L}(t) \\ = \min(t^L, N^L) & = \min(t, N^L) \end{array}$$

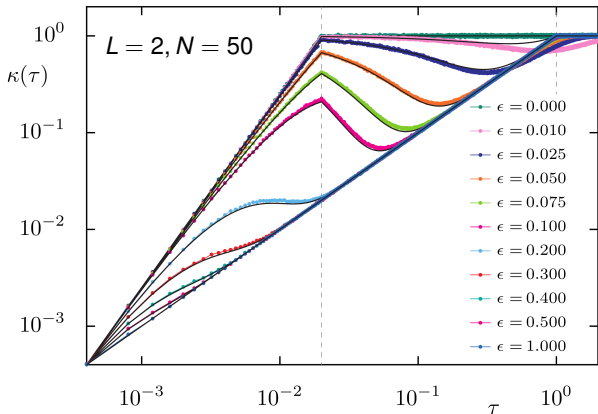


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$$0 \quad \quad \quad \sim 1 \quad \quad \rightarrow \sigma\epsilon$$

$$K(t) = (1 - |\chi(\epsilon)|^{2t})K_{N^L}(t) + |\chi(\epsilon)|^{2t}K_N(t)^L$$

$$\chi(\epsilon) = \left\langle \mathbf{e}^{i\epsilon\xi} \right\rangle_{\xi}$$



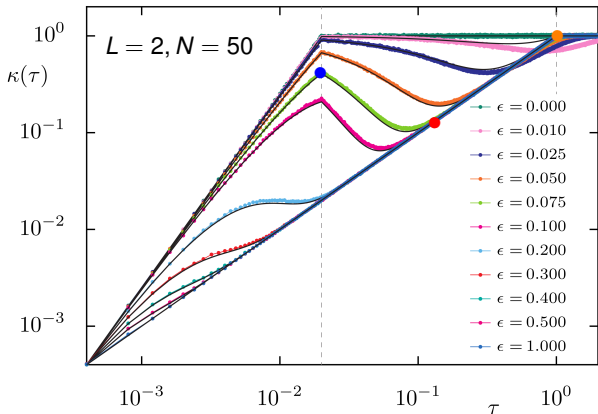
$$\kappa(\tau) = K(t)/N^L$$
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$$\kappa(\tau) = K(t)/N^L$$

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Time scales:

● Heisenberg time

$$\tau_H = 1$$

● ... of subsystems

$$\tau_{SH} = \frac{1}{N^{L-1}}$$

● Thouless time

$$t_{Th} = \frac{L \ln(N)}{2 |\ln |\chi(\epsilon)||}$$

# Exact SFF in the Semiclassical Limit [Chan, De Luca, Chalker, PRL (2018)]

- Factorization of averages over  $\text{CUE}(N)_A$ ,  $\text{CUE}(N)_B$ ,  $\xi_{ij}$ :

$$K(t) = \sum_{ijkl} \left( \begin{array}{c} U_A \\ \bullet \\ i_1 \\ \bullet \\ i_2 \\ \bullet \\ i_3 \\ \vdots \\ \bullet \\ i_t \end{array} \right) \left( \begin{array}{c} U_A^* \\ \bullet \\ j_1 \\ \bullet \\ j_2 \\ \bullet \\ j_3 \\ \vdots \\ \bullet \\ j_t \end{array} \right) \left( \begin{array}{c} U_B \\ \bullet \\ k_1 \\ \bullet \\ k_2 \\ \bullet \\ k_3 \\ \vdots \\ \bullet \\ k_t \end{array} \right) \left( \begin{array}{c} U_B^* \\ \bullet \\ l_1 \\ \bullet \\ l_2 \\ \bullet \\ l_3 \\ \vdots \\ \bullet \\ l_t \end{array} \right) \left\langle \exp \left( i\epsilon \sum_{ab} \xi_{ia} k_b - \xi_{jalb} \right) \right\rangle_{\xi}$$

$\bullet \text{---} \bullet = U_{mn}$

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The diagram illustrates the factorization of the spectral form factor  $K(t)$  into four parts, each representing a trace of a unitary matrix or its adjoint, and an average over disorder parameters  $\xi$ .

- Diagram 1 (Pink):** A vertical rectangle labeled  $U_A$  at the top and bottom. Inside, a solid black line forms a loop connecting points  $i_1, i_2, i_3, \dots, i_t$  on the left side.
- Diagram 2 (Light Blue):** A vertical rectangle labeled  $U_A^*$  at the top and bottom. Inside, a dashed black line forms a loop connecting points  $j_1, j_2, j_3, \dots, j_t$  on the left side.
- Diagram 3 (Pink):** A vertical rectangle labeled  $U_B$  at the top and bottom. Inside, a solid black line forms a loop connecting points  $k_1, k_2, k_3, \dots, k_t$  on the left side.
- Diagram 4 (Light Blue):** A vertical rectangle labeled  $U_B^*$  at the top and bottom. Inside, a dashed black line forms a loop connecting points  $l_1, l_2, l_3, \dots, l_t$  on the left side.

To the right of the diagrams, there are two boxes: a pink one containing  $\text{tr}(U^t)$  and a light blue one containing  $\text{tr}(U^{-t})$ . Below these is the average over disorder parameters  $\xi$  of the exponential of the sum of  $\xi_{i_a k_b} - \xi_{j_a l_b}$ .

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$$K(t) = \sum_{ijkl} \left( \begin{array}{c} U_A \\ \bullet i_1 \\ \bullet i_2 \\ \bullet i_3 \\ \vdots \\ \bullet i_t \end{array} \right) \left( \begin{array}{c} U_A^* \\ \bullet j_1 \\ \bullet j_2 \\ \bullet j_3 \\ \vdots \\ \bullet j_t \end{array} \right) \left( \begin{array}{c} U_B \\ \bullet k_1 \\ \bullet k_2 \\ \bullet k_3 \\ \vdots \\ \bullet k_t \end{array} \right) \left( \begin{array}{c} U_B^* \\ \bullet l_1 \\ \bullet l_2 \\ \bullet l_3 \\ \vdots \\ \bullet l_t \end{array} \right) \left\langle \exp \left( i\epsilon \sum_{ab} \xi_{ia} k_b - \xi_{jalb} \right) \right\rangle_{\xi}$$

$$\left\langle \cdot \right\rangle_{U_A} = \sum_{\mu, \nu \in S_t} \prod_a \delta_{i_a j_{\mu(a)}} \delta_{i_{a+1} j_{\nu(a+1)}} N^{-t - |\nu \mu^{-1}|} = \sum_{r=0}^{t-1} \prod_a \delta_{i_a j_{a+r}} N^{-t}$$

[Collins, Śniady, Commun. Math. Phys (2006)]

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# Universality of the Spectral Form Factor

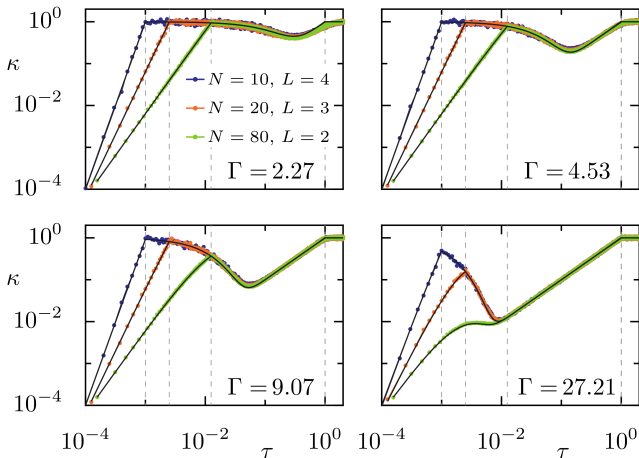
For large  $N$ :  $|\chi(\epsilon)|^{2t} \rightarrow e^{-\Gamma^2 \tau}$  with  $\Gamma = \sigma \epsilon N^{L/2}$

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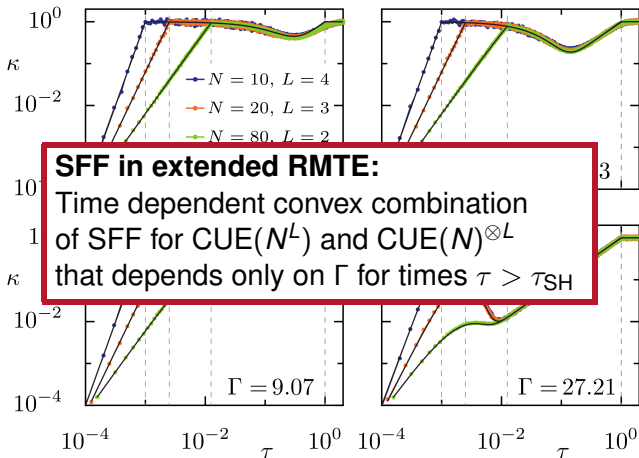
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▶ Moments:  $K^{(m)}(t) = \langle |\text{tr}(U^t)|^{2m} \rangle$

▶ CUE( $M$ ):  $K_M^{(m)}(t) = m! K_M(t)^m$

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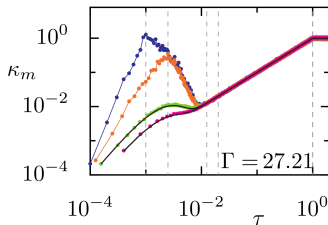
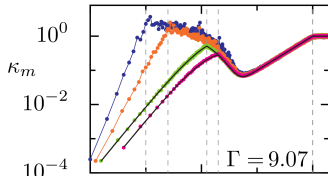
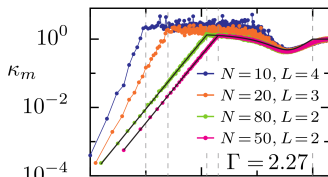
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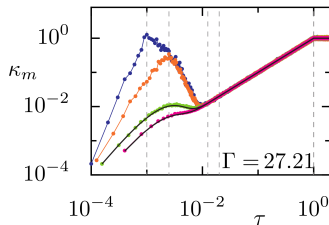
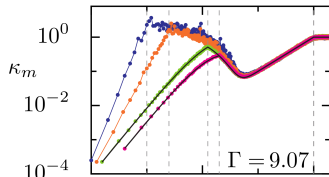
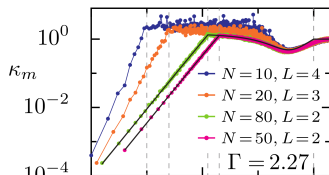
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# Coupled Kicked Rotors

- ▶ Periodically kicked rotors on 4-torus: [\[Froeschlé, Astron. & Astrophys. \(1972\)\]](#)

$$H(t) = \frac{p_A^2 + p_B^2}{2} + \frac{1}{4\pi^2} \sum_n \delta(t - n) \left[ k_A \cos(2\pi q_A) + k_B \cos(2\pi q_B) + \gamma \cos(2\pi [q_A + q_B]) \right]$$

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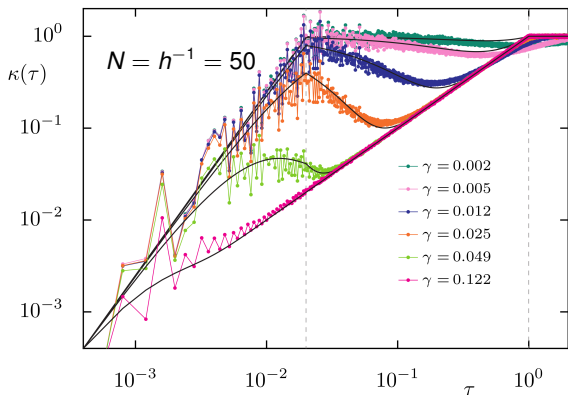
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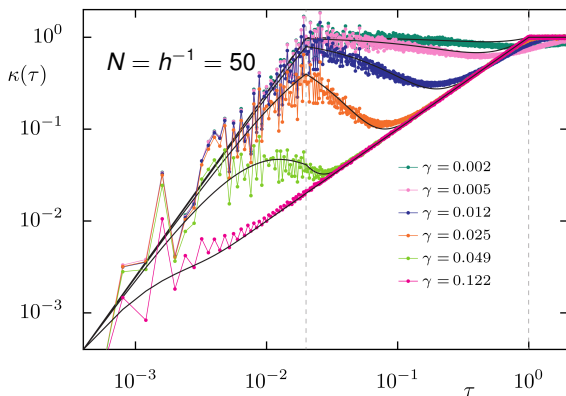
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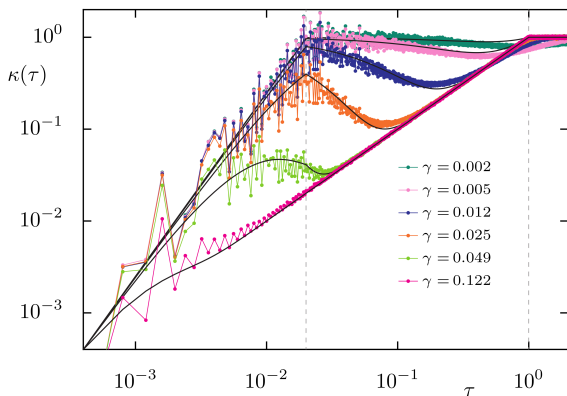
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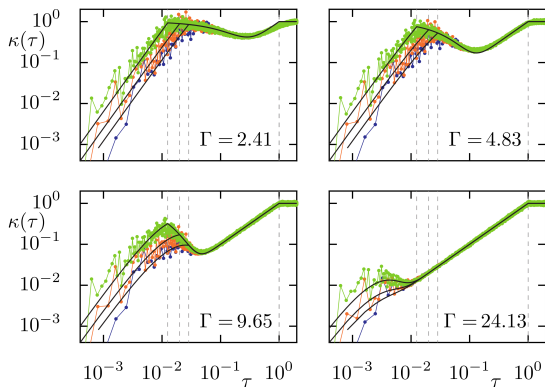
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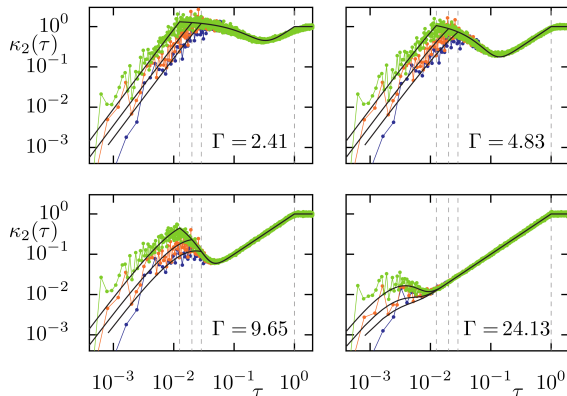
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# Summary

- ▶ Extended RMTE as minimal model for interacting few/many-body systems
- ▶ SFF and its moments in RMTE:  
Time dependent convex combination of noninteracting and strongly interacting limits
- ▶ Universal dependence of SFF on single scaling parameter  $\Gamma$
- ▶ Exact results for short times  $t < N$
- ▶ Applicable to quantized dynamical systems

More details: **arXiv.2302.09955**