Systems w/ long-range interactions

$$H = -\frac{J}{N(\alpha)} \sum_{i,j}^{N} \frac{J_{ij}\, \sigma_i^z \sigma_j^z}{|i-j|^\alpha} + h \sum_i^N \sigma_i^x$$

$\alpha \to \infty$: short-range interacting
$\alpha \to 0$: long-range interacting

① Extensivity

"Kac factor" $N(\alpha) = \frac{1}{N} \sum_{i \neq j} \frac{1}{D\, |i-j|^\alpha}$

$$N(\alpha) = \begin{cases} O(1) & \alpha > 1 \quad \rightsquigarrow \sim \infty \\ O(\log N) & \alpha = 1 \\ O(N^{1-\alpha}) & \alpha < 1 \end{cases}$$

$\alpha > 1$: ETH
$\alpha < 1$: Collective spin dynamics

→ limit $\alpha = 0$: Lipkin-Meshkov-Glik model

$$H = -\frac{J}{N} \sum_{i,j} \sigma_i^z \sigma_j^z + h \sum_i \sigma_i^x$$

⇝ Collective spin operator

$$S_{tot}^\alpha = \sum_{i=1}^N \sigma_i^\alpha / 2$$

⇝ $H = -\frac{J}{N} (S_{tot}^z)^2 + h\, S_{tot}^x$

⇝ conserves total spin
symmetric (& integrable) [⇝ all spin identical]
⇝ Schwinger boson mapping

⇝ Fix $S^{tot} = S$ *
⇝ Diagonalize $(2S+1)\times(2S+1)$ matrix in basis $\{|S, M_S\rangle\}$ of multiplets
[= Dicke states]

$\alpha < 1$: perturbation on top of this limit

⇝ ETH breaks down
⇝ dynamics = coherent oscillations

* why $\alpha = 1$: prethermalization & quasi-conservation of each symm

$\hat{P}_{ij}$ exchanges spin i & j

$$\hat{P}_{ij} = \frac{1}{2}(\mathbb{1}_{ij} + \vec{\sigma}_i \cdot \vec{\sigma}_j)$$

$$|\langle \hat{P}_{ij}(t)\rangle - \langle P_{ij}(0)\rangle| \leq C_\alpha \frac{t}{N^{1-\alpha}} J_{ij}$$

Mori, JPA 52 054001 (2019)

$$\frac{d}{dt}\langle \hat{P}_{ij}(t)\rangle = -i\langle [\hat{P}_{ij}, H]\rangle$$

$$= -i\langle P_{ij} H - H P_{ij}\rangle$$

$$= -i\langle P_{ij}(H - P_{ij} H P_{ij})\rangle$$

$$\leadsto |\langle P_{ij}(t)\rangle| \leq t\, \| P_{ij}(H - P_{ij} H P_{ij})\|$$

$\leq t \, \| H - P_{ij} H P_{ij} \|$

②

$P_{ij} H P_{ij} = H$ with $i \,\&\, j$ exchanged

$$H - P_{ij} H P_{ij} = - \sum_{k \neq i,j} (J_{ik} - J_{jk}) (\sigma_i^z - \sigma_j^z) \sigma_k^z$$

$$\Rightarrow \| H - P_{ij} H P_{ij} \| \leq \cancel{\frac{1}{N^{1-\alpha}} \times} \sum |J_{ik} - J_{jk}|$$

$$\leq 2 \sum_{k \neq i,j} |J_{ik} - J_{jk}| \leq \frac{1}{N^{1-\alpha}} C_\alpha$$

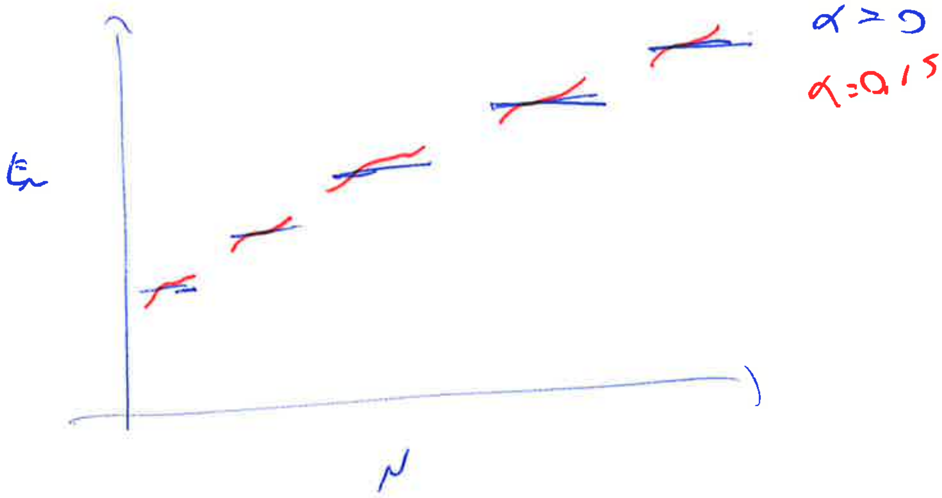
$$\Rightarrow |\langle P_{ij}(t) \rangle - \langle P_{ij}(0) \rangle| \leq C_\alpha t / N^{1-\alpha} + J_{ij}$$

$\Rightarrow$ symmetric until time scales as $N^{1-\alpha}$

$\Rightarrow$ can restrict to Dicke manifold for increasing long times

"spin wave"

ETH? Russomanno et al. PRB 104, 094309 (2021)



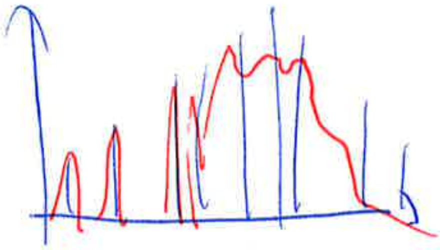
Assume that GOE within multiplets

⇝ broaden ~ α N

⇝ eigenvalue spectra in multiplets ~ Wigner's semicircle law

⇝ multiplets with asymptotically small level spacing

⇝ some multiplets survive



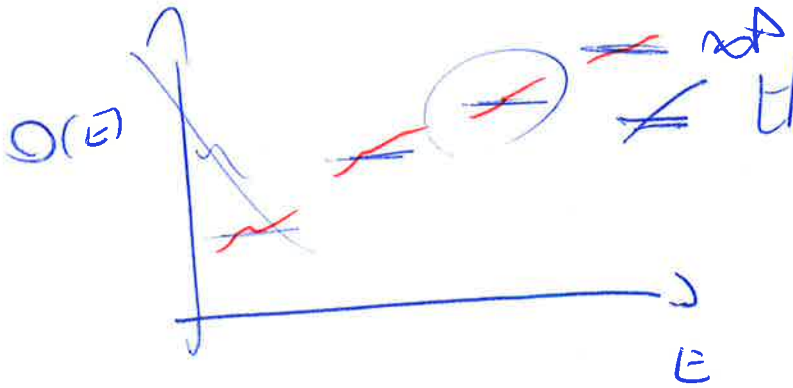
→ Low-lying multiplets survive!

→ & S maximal

E

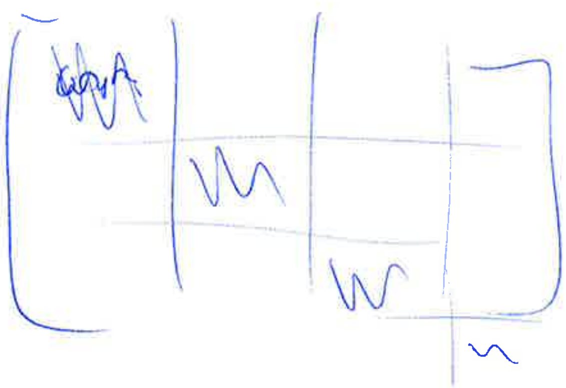
nonconvex = inequivalence of ensembles

⇝ ETH fails



≠ thermal value

$$w \sim \alpha\, 2^{N/2} K \sqrt{\frac{g^{(S)}}{N}}$$



Alternative (Nicolò Defenu, PNAS, 'Metastability of discrete spectrum...')

$$H = -\sum_{i,r} \frac{1}{N_\alpha}\frac{1}{r^\alpha}\sum (a_i^\dagger a_{i+r} + h.c.) + \mu \sum a_i^\dagger a_i \qquad \Big| \quad t_r = \frac{1}{N_\alpha}\frac{1}{r^\alpha}$$

⇝ Go to momentum space

$$H_0 = -\sum \varepsilon(k)\, a_k^\dagger a_k$$

with $\varepsilon(k) = \mu - \tilde{t}_k$ with $\tilde{t}_k = \frac{1}{N_\alpha}\sum \frac{\cos(kr)}{r^\alpha}$

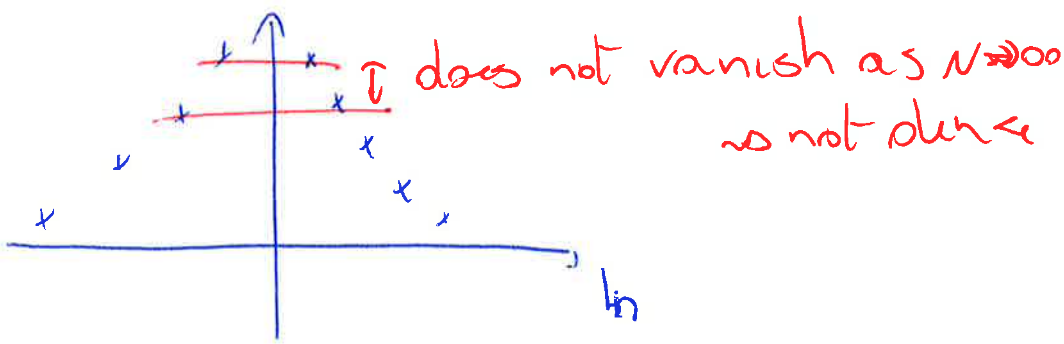
⇝ $\tilde{t}_k \approx c_\alpha \int_0^{1/2} \frac{\cos(2\pi n s)}{s^\alpha}\, ds$ with $k = \frac{2\pi n}{N}$

with $c_\alpha = (1-\alpha)\, 2^{1-\alpha}$

⇝ $\alpha = 0$: exactly deg.: $\tilde{t}_k = 0$

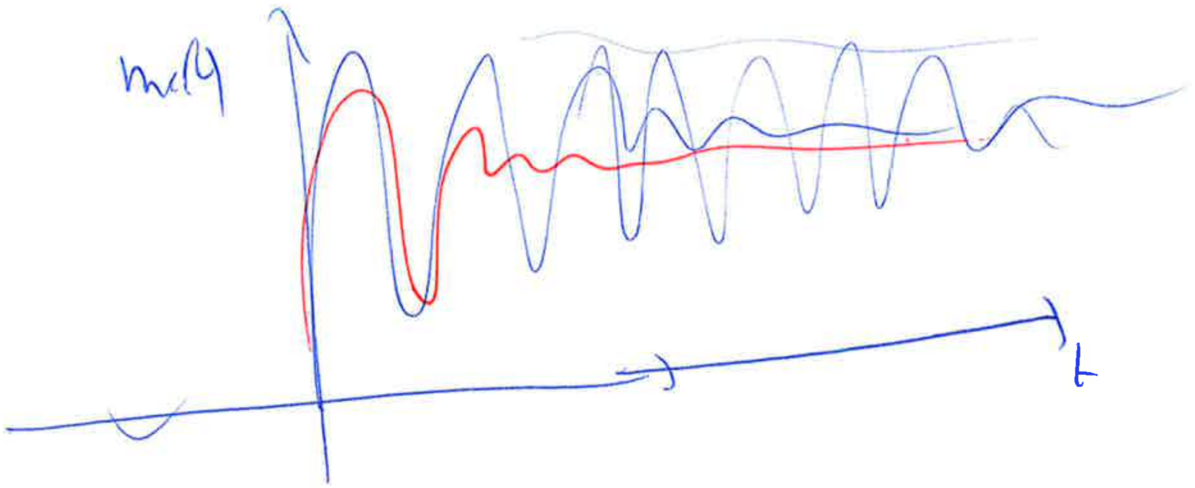
⇝ Discrete spectrum!

.. for $\alpha < 1$



→ Dynamics = oscillating

[physically: system ~ penetration depth
which are ~~under[illegible]~~ finite length scale,
observables cannot resolve fine wavelengths
→ only modes w/ $k \sim \pi/L$ can capture dynamics



Heisenberg time

$\rightsquigarrow$ semiclassical limit

$|S, m_S\rangle \Rightarrow |q\rangle$, $Nq \in [-N/2, \dots N/2]$

large N $\rightsquigarrow$ continuum limit

$$p = \frac{-i}{N}\frac{\partial}{\partial q} \rightsquigarrow \frac{i}{N}\frac{\partial}{\partial t}\psi_t(q) = H\,\hat{\psi}_t(q)$$

$$H \approx -2q^2 - 2h_z q - h_x\sqrt{1-(2q)^2}\cos p$$

$\rightsquigarrow$ Effective Planck's constant

$$\hbar \sim 1/N$$

$\rightsquigarrow$ Ehrenfest time $\sim \log N$