Quantum Many-Body Scars in DU Circuits

PRL 132, 0140401

Refs:

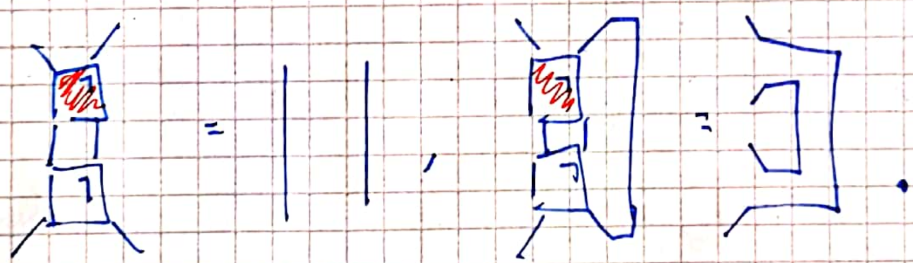
I • PRL 129, 02061 (2022)

II • PRX 2, 020330 (2021)

III • PRB 107, 035123 (2023)

IV • Rep. Prog. Phys. 85, 086501 (2022)

Quick recap on DU circuits:



For qubits one has

$$U^{DU} \equiv e^{i\phi}(u_+ \otimes u_-)\, V[J_x, J_z, J_y]\, (v_- \otimes v_+)$$

with

$$J_x = J_y = \frac{\pi}{4}.$$

See PRL 125, 07051 (2020) & PRA 69, 032315.

"Scars are basically ETH-violating eigstates which decouple from the thermal bulk".

In this paper they build a SU(2)-like algebra from "ladder operators" which are dynamical symmetries of the Hamiltonian:

$$([\hat{H}, \hat{Q}^\dagger] - \omega \hat{Q}^\dagger) W = 0.$$

Model dependent

Linear subspace

In this paper they do this for the PXP model.

Applications: generation of (stable) multipartite entanglement, metrology.

$$\langle E_m | \hat{O} | E_n \rangle = \bar{O}(\bar{E}) \delta_{m,n} + R_{m,n} f_O(\bar{E}, \omega) / \sqrt{\Omega(\bar{E})}.$$

$\omega = E_m - E_n$

$\bar{E} = (E_m + E_n)/2$

Overall, there is a bunch of techniques to embed QMBS in your model. E.g., the Shiraishi-Mori formalisms. So you have projector based approaches, symmetry based, etc. See Chapter 3, especially 3.2 of Ref IV for different methods. The notion of tower of states.

PRL 119 030601 (2017)

QMBS in DU circuits.

Main message: there is a projector-based formalism to introduce QMBS/weak ergodicity breaking in DU circuits.

Dual-unitarity means:

$\langle k| \otimes \langle l| \, \tilde{U} \, |i\rangle \otimes |j\rangle$

$=$

$\langle j| \otimes \langle l| \, U \, |i\rangle \otimes |k\rangle$, w/ both U and $\tilde{U}$ unitary.

$\tilde{U}$ being the dual of U.

In general:

$$\hat{U}^{DU,1} = (u^+ \otimes u^-) \, S \, V \, (v^- \otimes v^+)$$

is DU. Here S is the SWAP.

and

$$\hat{V} = \exp\left\{ i \sum_{j=0}^{d-1} \hat{h}^{(j)} \otimes |j\rangle\langle j| \right\}.$$

To move ~~on~~ forward, we introduce the convenient notation:

$$u^+ \otimes u^- \equiv \exp\{ i (\hat{f}^+ \otimes I + I \otimes \hat{f}^-) \},$$

$$v^- \otimes v^+ \equiv \exp\{ i (\hat{g}^- \otimes I + I \otimes \hat{g}^+) \}.$$

Why?

DU gate specified by $\{\hat{f}^{\pm}, \hat{g}^{\pm}, \hat{h}^{(j)}\}$

The scars will be built on top of these single-qudit gates.

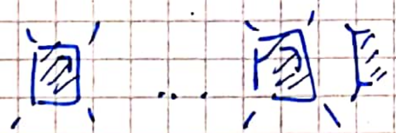
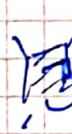
Also, note that

$\hat{U}^{DU,2} = \hat{S}\hat{U}^{DU,1}\hat{S}$ is also DU.

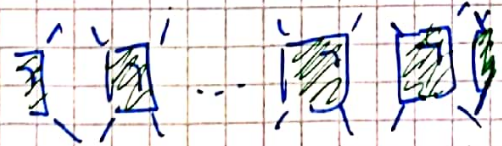
In general, does not fall under previous parametrization.

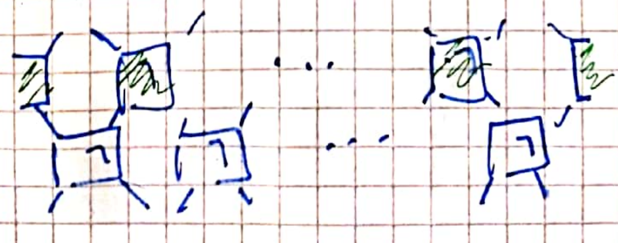
Let us construct the circuit.

Take:

$U_e =$  , w/  $:= \hat{U}^{DU,1}$

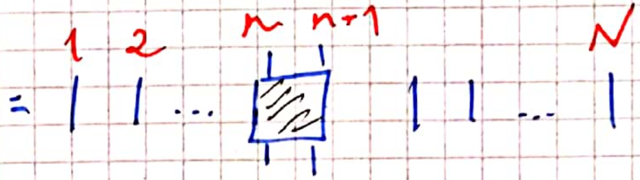
but

$U_o =$  w/  $:= \hat{S}\hat{U}^{DU,1}\hat{S}$.

$\Rightarrow$ $U :=$ 

Idea: use projectors associated w/ generators $\{\hat{f}^{\pm}, \hat{g}^{\pm}, \hat{h}\}$ to obtain a subspace w/ trivial dynamics.

Define:

$\hat{P}_{n,n+1} =$ 

with  $:= P_{n,n+1}$. $\hat{P}$ is the extended projector.

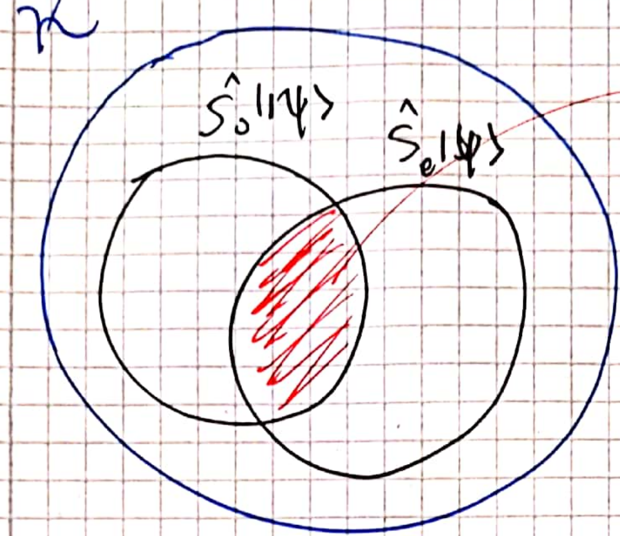
Common Kernel: set of $|\psi\rangle$ annihilated by all projectors.

$$\mathcal{K} = \{ |\psi\rangle : \hat{P}_{n,n+1} |\psi\rangle = 0, \ \forall n \}$$

Target states: Set $\mathcal{T}$ s.t.

$$\mathcal{T} = \{ |\psi\rangle : |\psi\rangle \in \mathcal{K}, \hat{S}_e |\psi\rangle \in \mathcal{K}, \hat{S}_o |\psi\rangle \in \mathcal{K} \}.$$

Subset of $\mathcal{K}$ invariant to even odd swap.



Target states in set T

$\Downarrow$

This means that $\hat{U}$ just permutes states in T!

(I.e., $\hat{U}|\psi\rangle = \hat{S}_o \hat{S}_e |\psi\rangle \in T$)

where

$$\hat{S}_e = X \; X \; \dots \; X$$

$$\hat{S}_o = X \; X \; \dots \; X \; X \; \dots$$

Why define this?

- Impose conditions on $\{\hat{f}^{\pm}, \hat{g}^{\pm}, \hat{h}^{(j)}\}$
- Generators of $U^+ \otimes U^-$, $V^+ \otimes V^-$ & $\hat{V}$ are invariant under action of projections!

$$\boxed{\hat{P}_{n,n+1} \left(\hat{f}_n^{+} \otimes I + I \otimes \hat{f}_{n+1}^{-}\right) \hat{P}_{n,n+1} \overset{!}{=} \hat{f}_n^{+} \otimes I + I \otimes \hat{f}_{n+1}^{-}}$$

This means that

$v^+ \otimes v^-$, $v^+ \otimes v^- \otimes V$ act

trivially on $|\psi\rangle \in K$. Kernel!

However, note that

$\hat{U}^{D_{n,1}}$ includes a SWAP $\hat{S}_{n,n+1}$

in its definition, so even

w/ the conditions above

$\hat{U}$ might still take $|\psi\rangle$

out of K. That is

why we consider $T \subset K$,

we need this extra

condition.

How to do this in practice?

Construction of Scars

Consider projectors of the type

$$\hat{P}_{n,n+1} = \mathbb{I}_{n,n+1} - \sum_{x \in \mathcal{X}} |x\rangle\langle x|_{n,n+1}$$

w/ $|x\rangle_{n,n+1} := |x'\rangle_n \otimes |x''\rangle_{n+1}$.

The first two conditions on $\hat{f}$ and $\hat{g}$, are satisfied if

$\hat{f}^{+}|x'\rangle = \hat{g}^{-}|x'\rangle = 0$

$\hat{f}^{-}|x''\rangle = \hat{g}^{+}|x''\rangle = 0 \quad \forall\ |x\rangle \in \mathcal{X}$

(I.e., certain rows & columns are 0).

Besides that, $\hat{f}$, $\hat{g}$ and $\hat{h}$ can be random. Finally, for $\hat{h}$ we also impose that

$$\hat{H}_{n,n+1} = \hat{P}_{n,n+1} \hat{\tilde{H}}_{n,n+1} \hat{P}_{n,n+1},$$

with $\hat{\tilde{H}}_{n,n+1} = \sum_{j=0}^{d-1} \hat{\tilde{h}}^{(j)} \otimes |j\rangle\langle j|$

and $V_{n,n+1} = \mathrm{dep}(\hat{H}_{n,n+1})$ given $\tilde{h}^{(j)}$ random.

The reason for this is that $\exp(\hat{\tilde{H}})_{n,n+1}$ by itself does not satisfies the necessary conditions, so it must be projected first.

Example: Take

$\chi = \{|0,0\rangle, |d-1, d-1\rangle, |0, d-1\rangle, |d-1, 0\rangle\}$.

The target space will be

$\mathcal{T} = \mathrm{span}\{|0\rangle, |d-1\rangle\}^{\otimes N}$

w/ $\dim(\mathcal{T}) = 2^N$. However note that $\left(\frac{2}{d}\right)^N \to 0$ as $N \to \infty$. This is still exponentially small.

The message here is that the very convenient parametrization of DU-circuits points us to a very simple ~~const~~ projector-based approach to embed QMBSs on them.

The resulting scars are non-trivial and even exponentially many.

Insightful results in terms of: half-system bipartite entanglement entropy

Important remark:

The states $|\vec{i}\rangle := |i_1, i_2, \ldots, i_N\rangle$ w/ $i_j \in \{0, d-1\}$ are not necessarily QMBS. The only "trivial" ones are: $|0\rangle^{\otimes N}$, $|d-1\rangle^N$, $|0, d-1\rangle^{\otimes N/2}$ and $|d-1, 0\rangle^{\otimes N/2}$.

One must diagonalizing the SWAP circuit $\hat{S} = \hat{S}_0 \hat{S}_e$ in span$\{|\vec{i}\rangle\}$. Why? Examples:

Fine

$$S_0 S_e |0, d-1, 0, d-1\rangle$$
$$= S_0 |d-1, 0, d-1, 0\rangle$$
$$= |0, d-1, 0, d-1\rangle \checkmark$$

Eigstate!

Not Fine

$$S_0 S_e |0, d-1, d-1, 0\rangle$$
$$= S_0 |d-1, 0, 0, d-1\rangle$$
$$= |d-1, 0, 0, d-1\rangle$$

Not Eigstate!

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