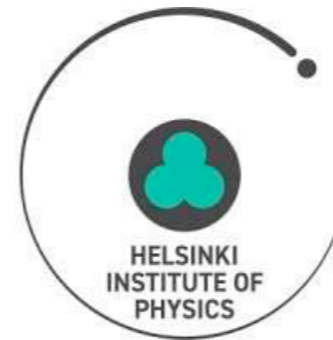


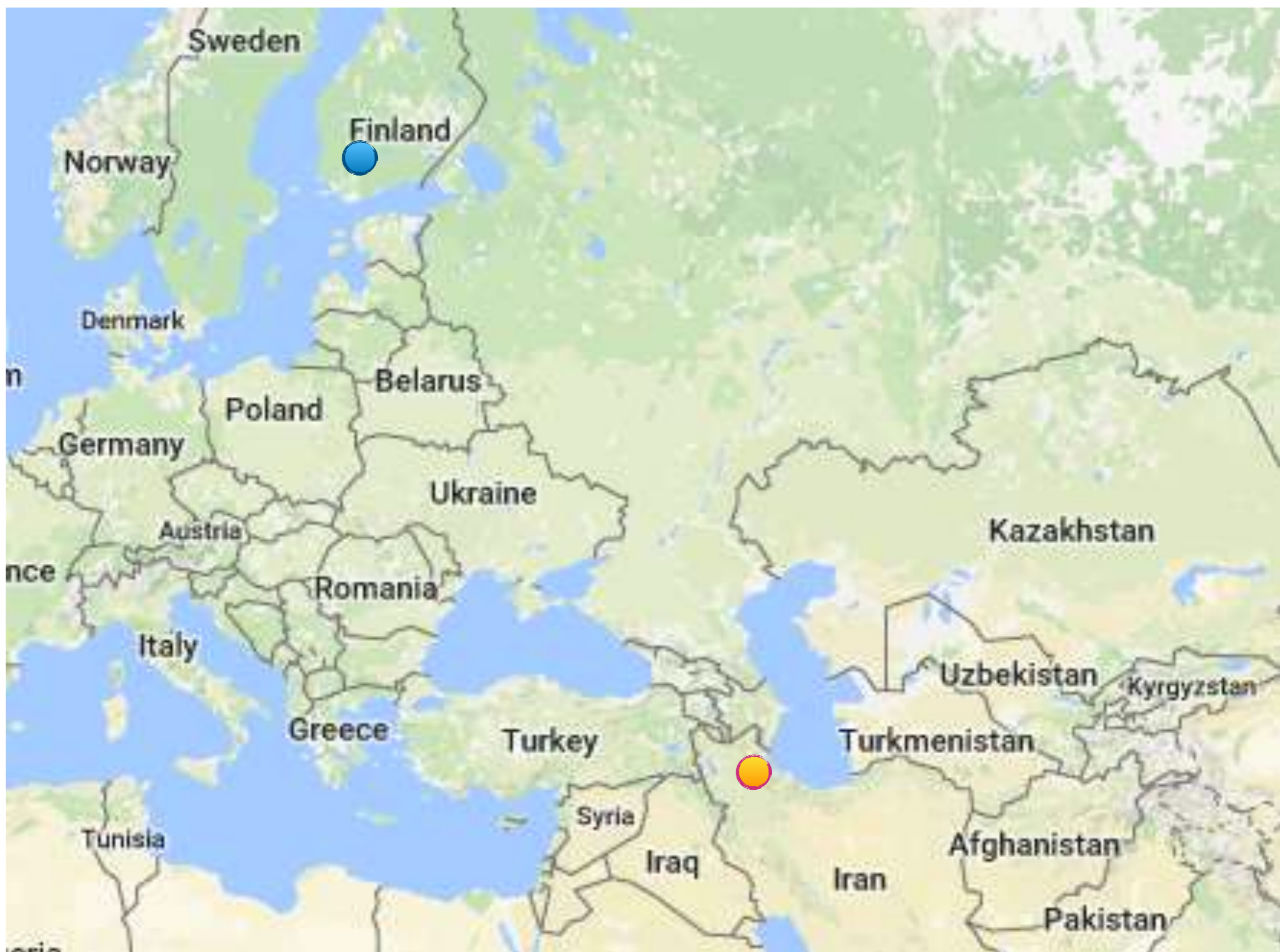
Scalable Approaches for Probing Phase Transitions in Symmetric Monitored Quantum Circuits

Ali G. Moghaddam



MPI-PKS

Dresden, July 2024



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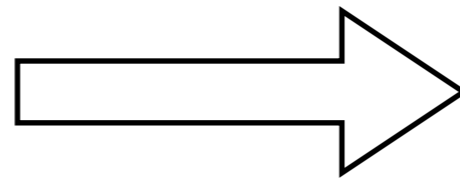


Outline

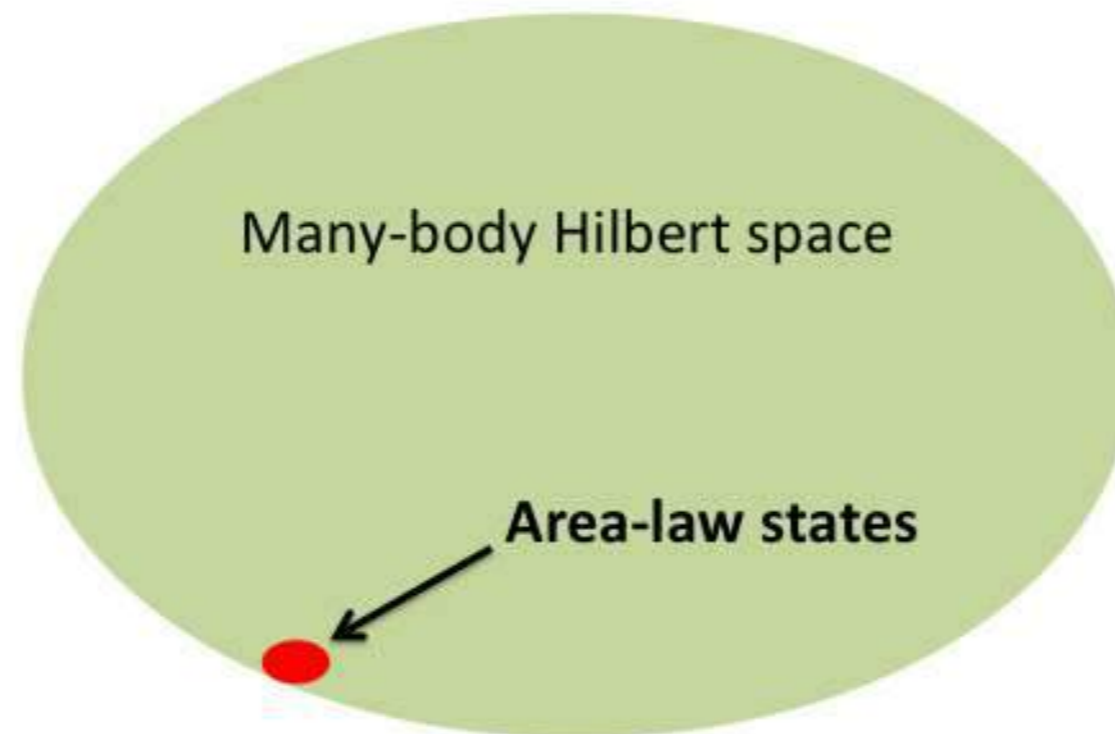
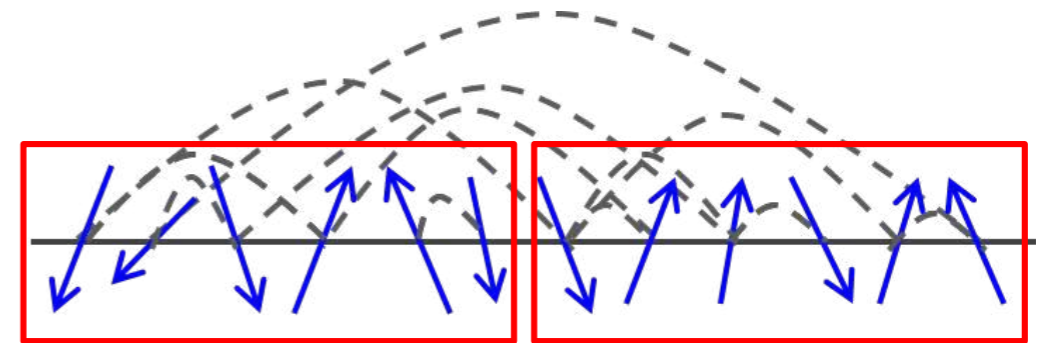
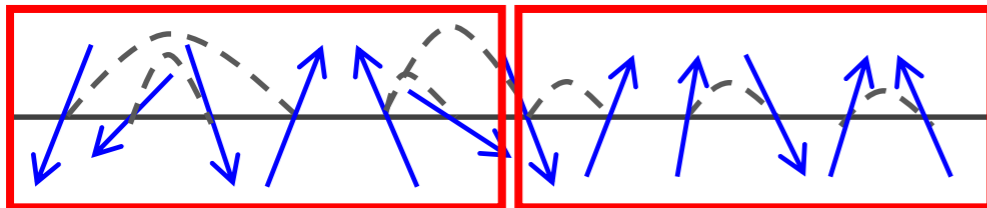
- Manybody entanglement, MIPT
- Connection between entanglement and fluctuations in the presence of conservation
- Bypassing tomography by probing fluctuations
- Combining with steering: avoidable postselection
- Estimating overhead; only polynomial!
- Summary

Manybody entanglement

Weakly entangled:
area-law

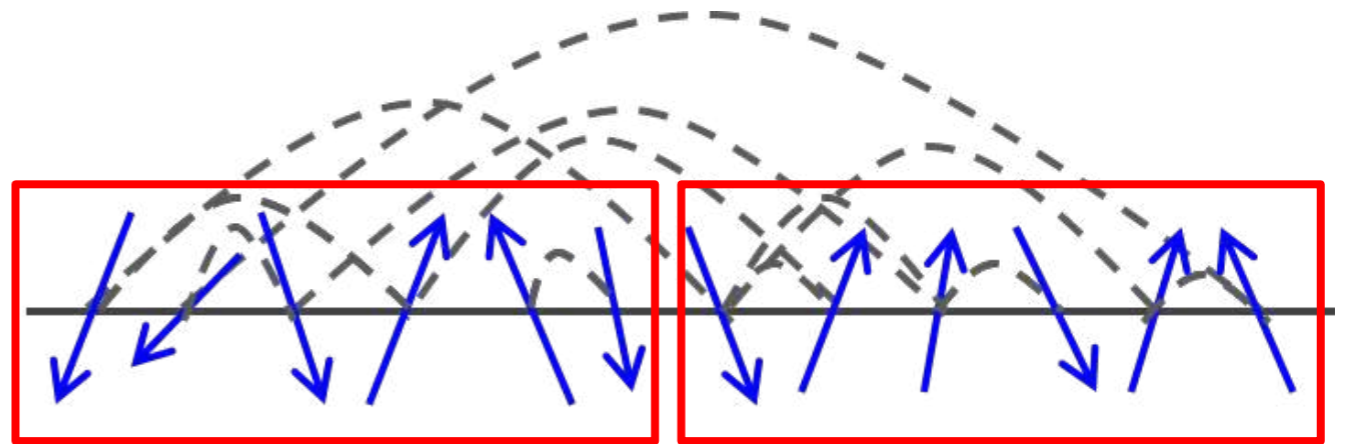


Highly entangled:
volume-law

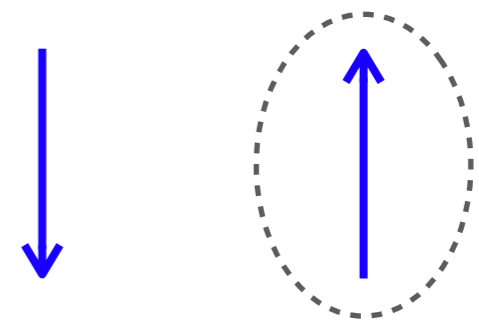
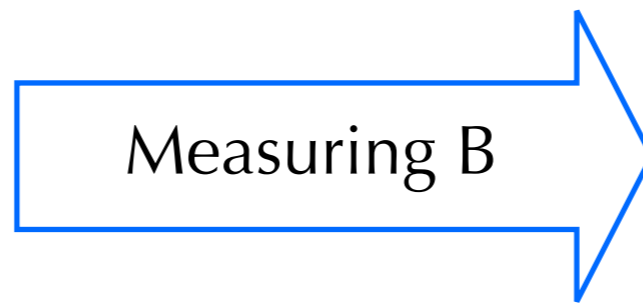
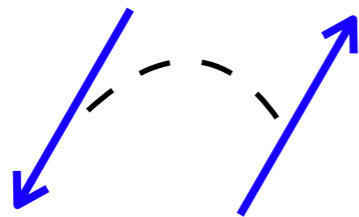
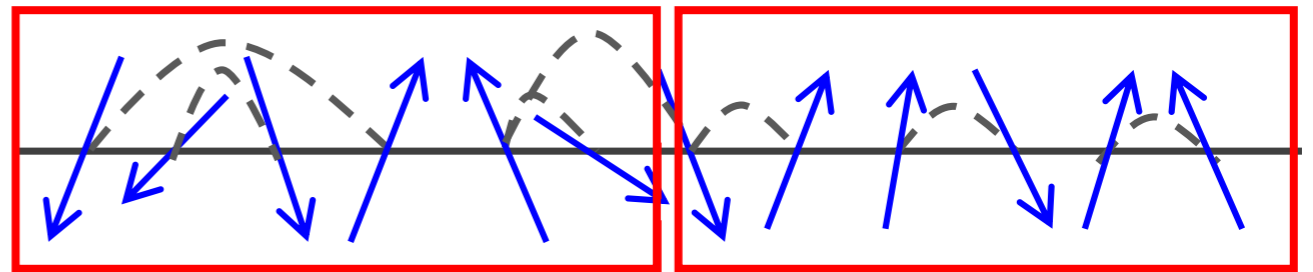


Entanglement vs measurement

Highly entangled:
volume-law



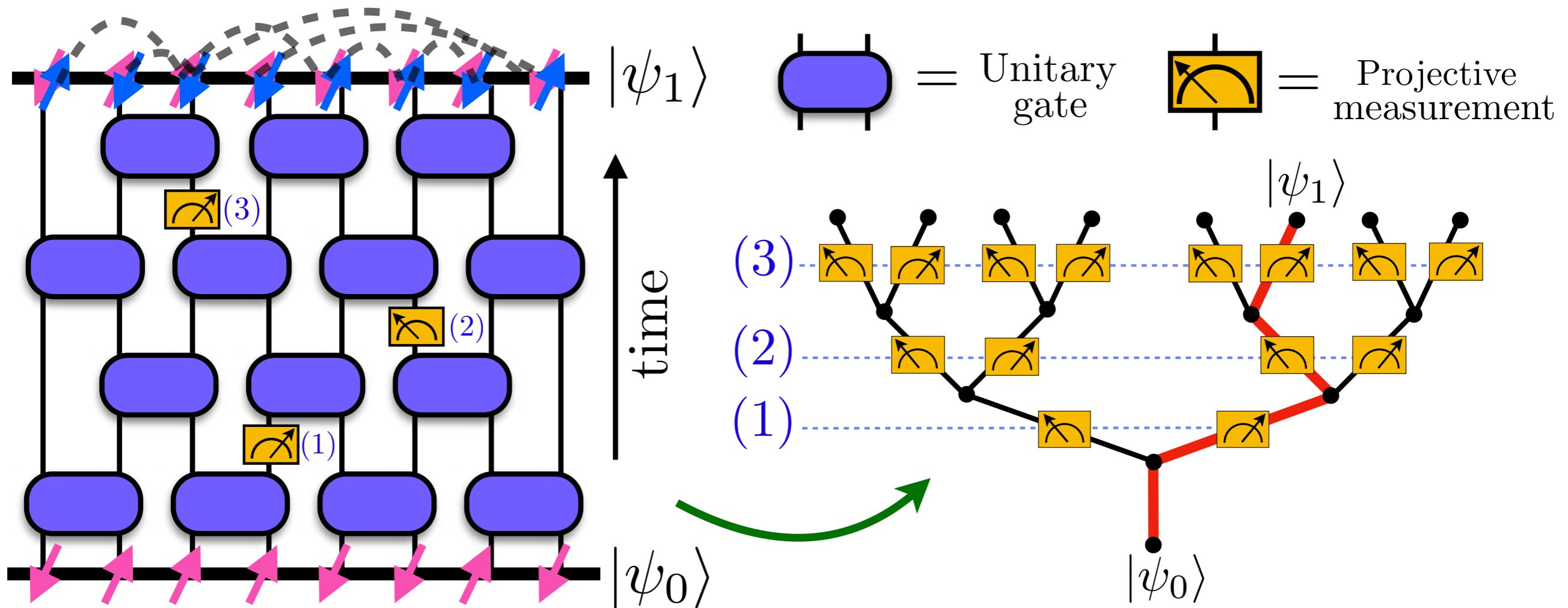
Weakly entangled:
area-law



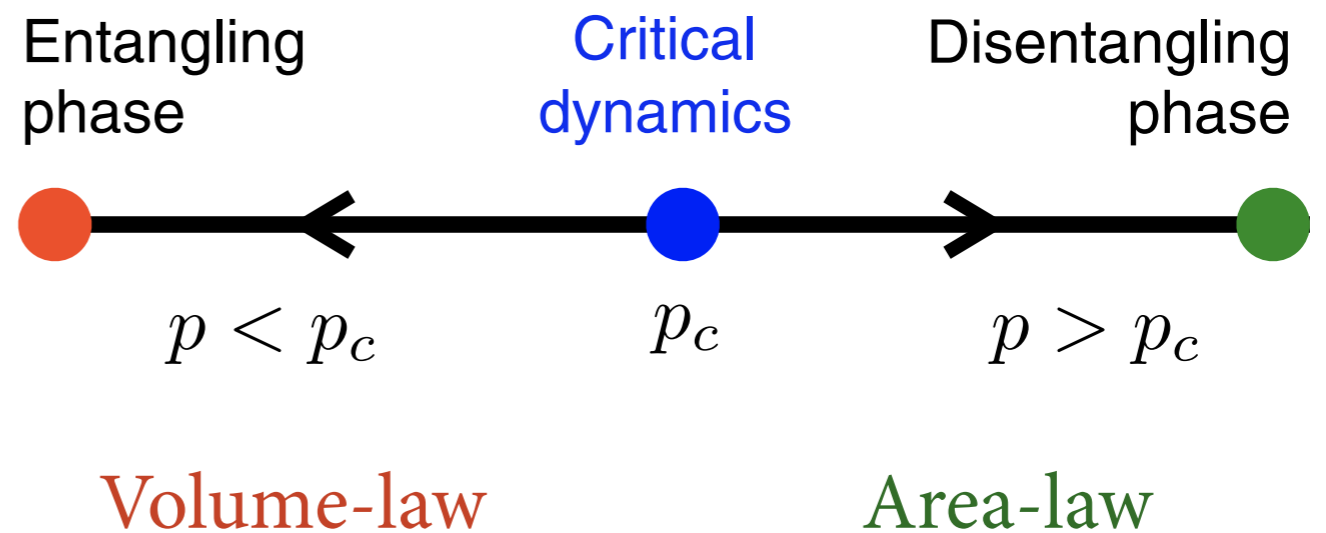
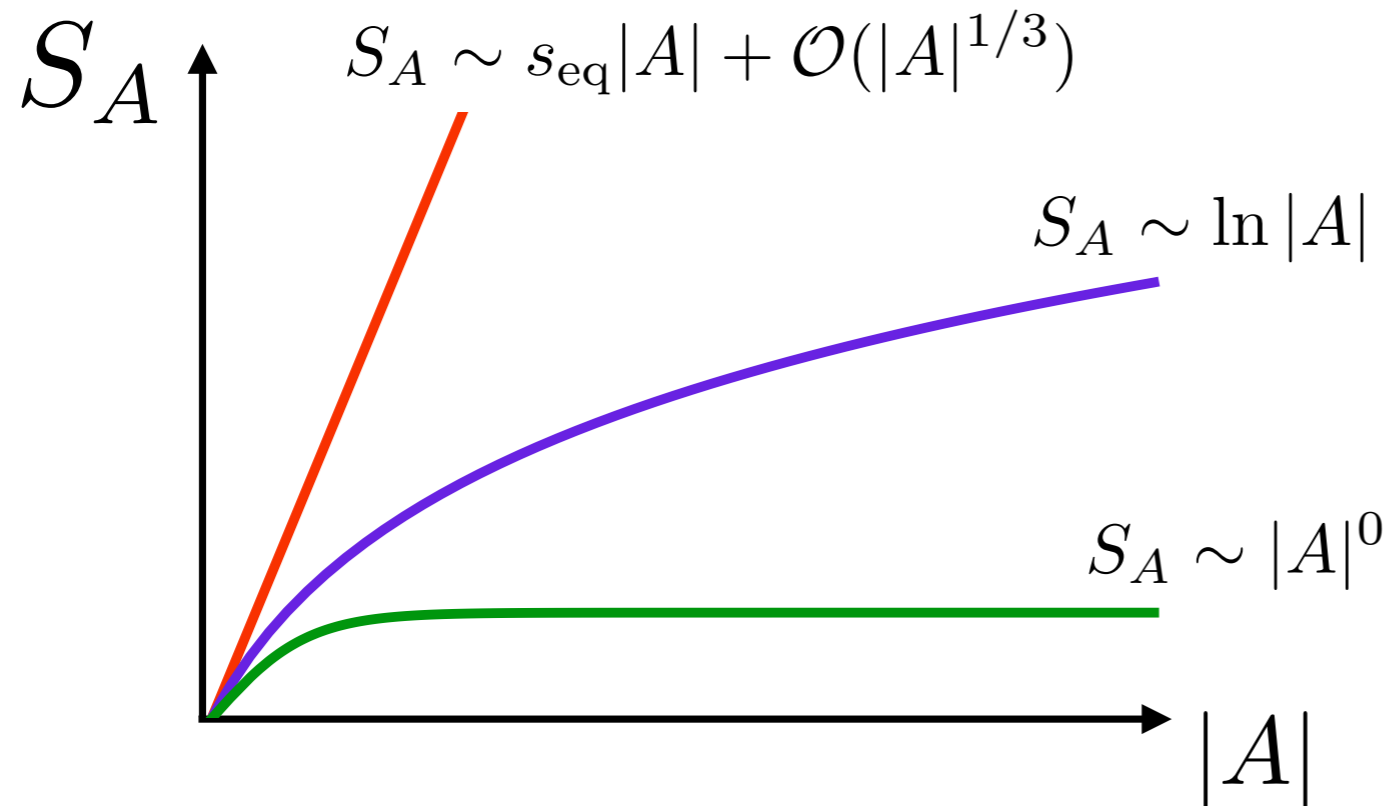
$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow_A \downarrow_B\rangle - |\downarrow_A \uparrow_B\rangle)$$

$$|\psi\rangle = |\downarrow_A \uparrow_B\rangle$$

Monitored quantum dynamics

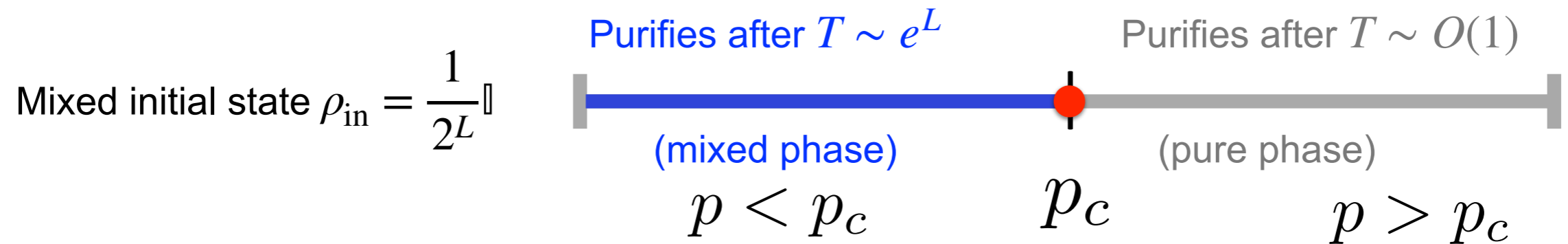


Measurement-induced phase transition (MIPT)



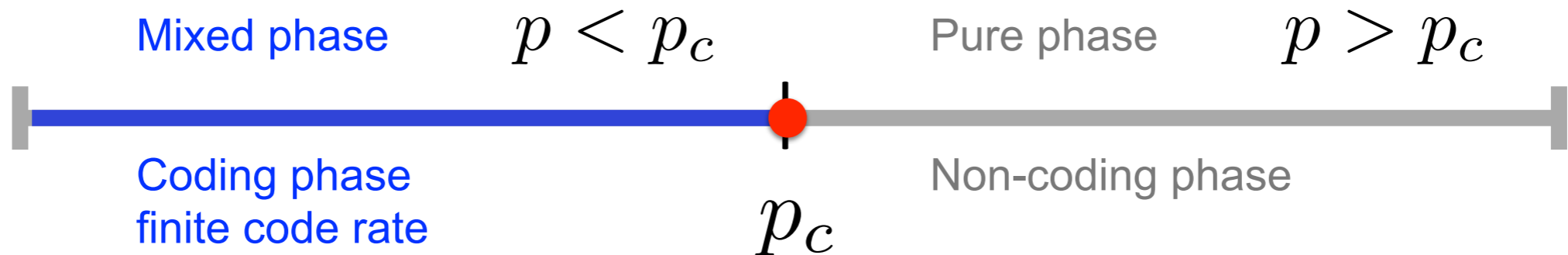
Other manifestaions of MIPT

Purification transition



Gullans, Huse, PRX (2019)
Noel et. al., Nat. Phys. (2022)

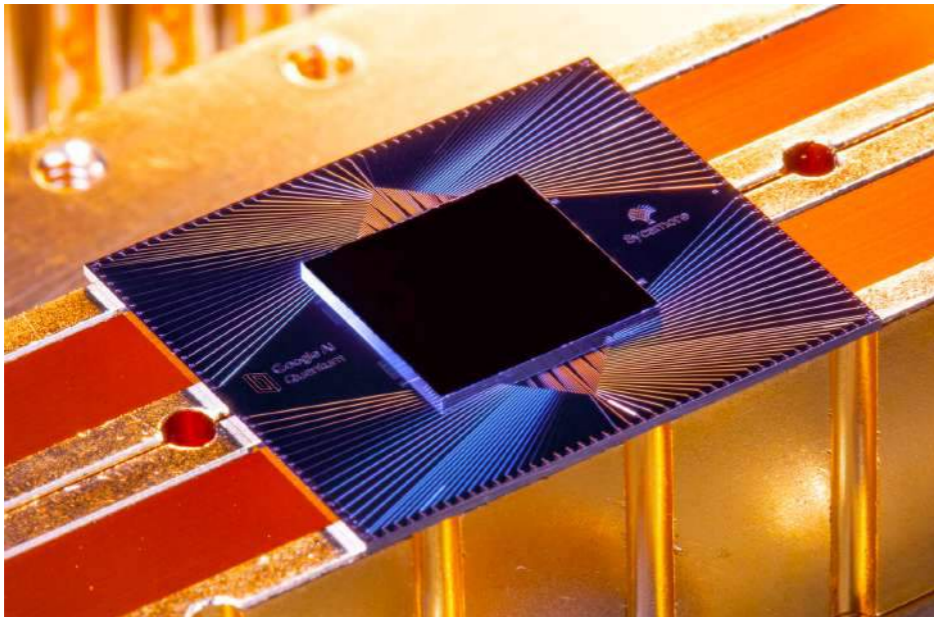
Dynamical quantum memory (dynamical QECC)



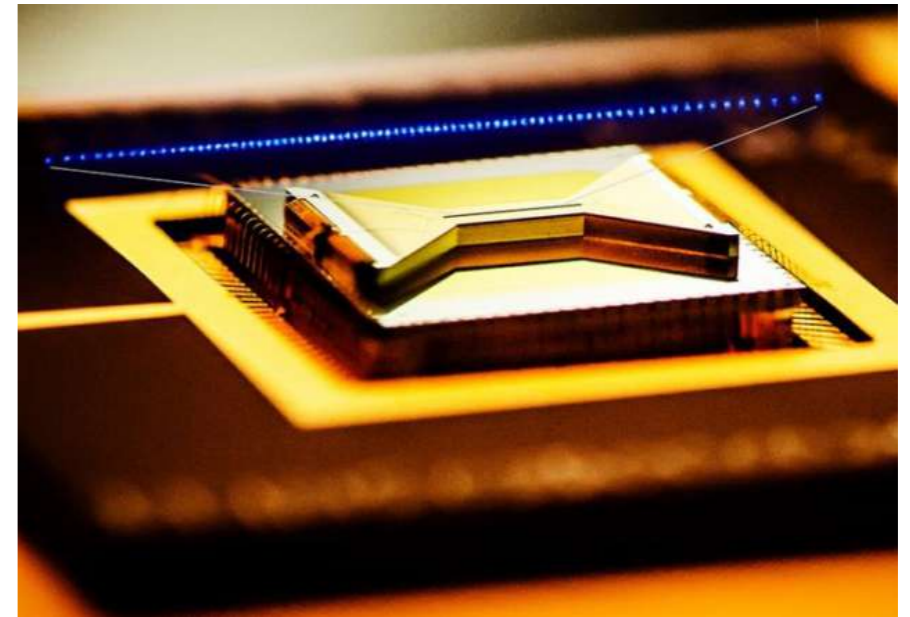
Hayden & Preksill, JHEP (2007)
Choi, Bao, Qi, Altman PRL (2019)

Recent experiments

Superconducting qubits
(IBM-Caltech; Google AI Quantum)



Trapped ions (IonQ)



Noel et al., Nat. Phys. (2022)

Probing purification transition
using single reference qubit

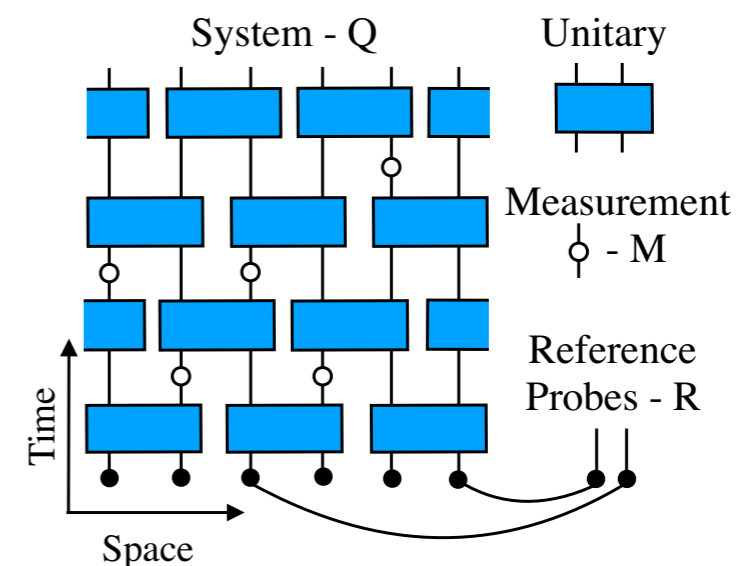
Full postselection and tomography

Hoke et al., (Google Quantum AI), Nature (2023);
Koh et al., Nat. Phys. (2023);

Utilizing cross-entropy benchmarking

Kamakari et al., arXiv:2403.00938.

Theory: Li et al., PRL (2023)



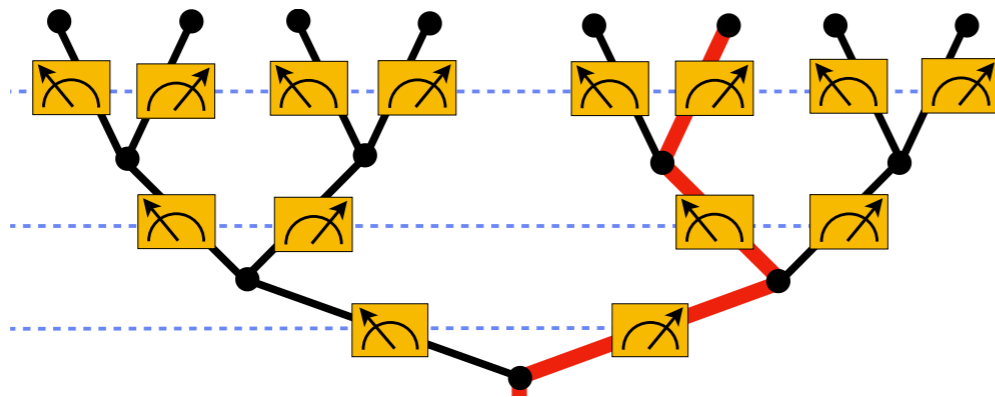
Theory: Gulans & Huse, PRL (2020)

Exponential complexity I: postselection

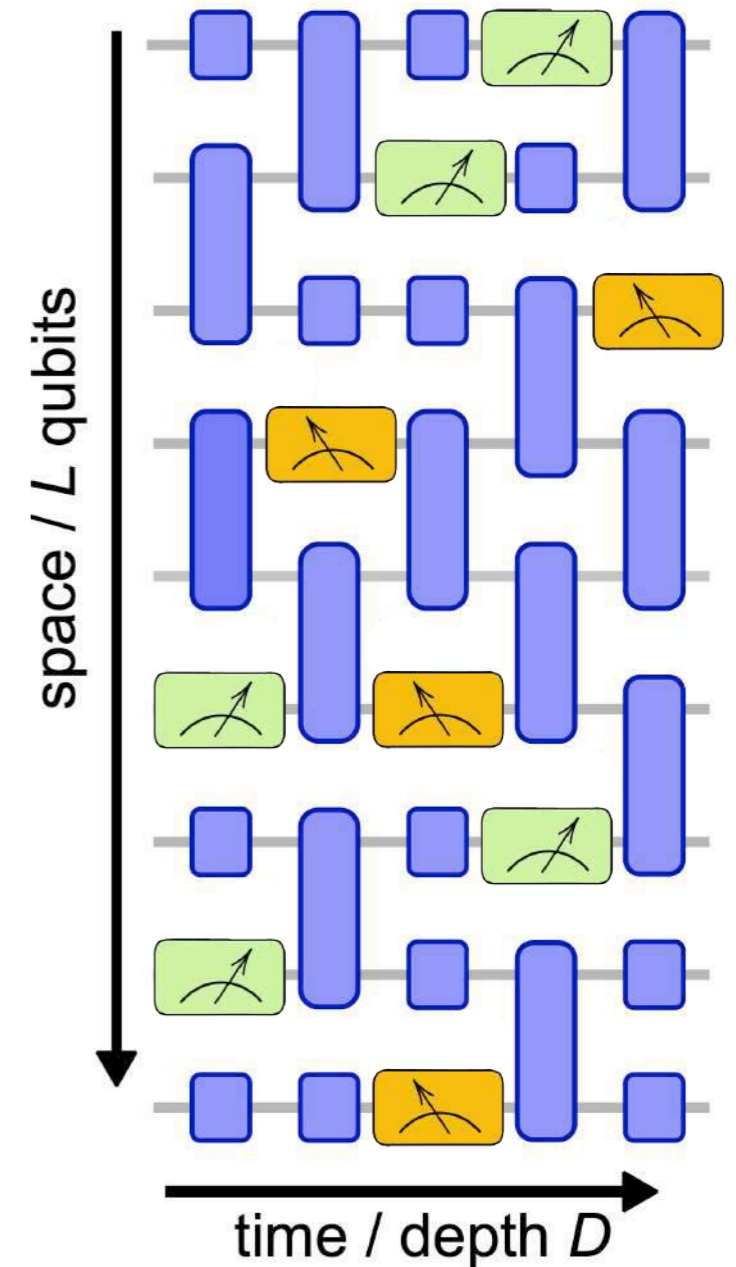
$$\langle S_A \rangle = \sum_{\mathbf{m}} p_{\mathbf{m}} S_A (|\psi_{\mathbf{m}}\rangle \langle \psi_{\mathbf{m}}|)$$

Average # of measured qubits $\|\mathbf{m}\| = pLT$

of quantum trajectories 2^{pLT}

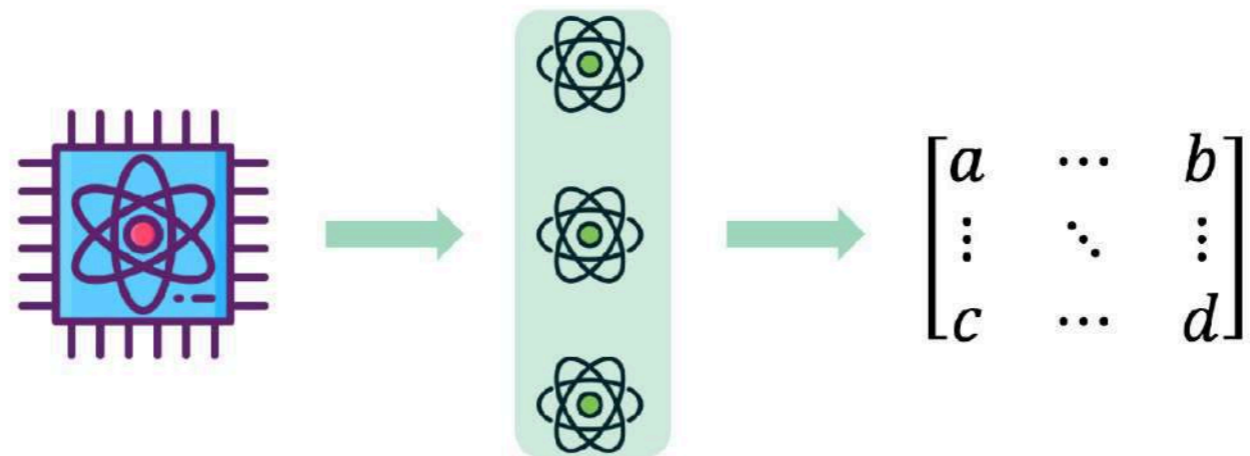


$$\mathbf{m} = (m_1, m_2, \dots, m_N)$$



Exponential complexity II: tomography

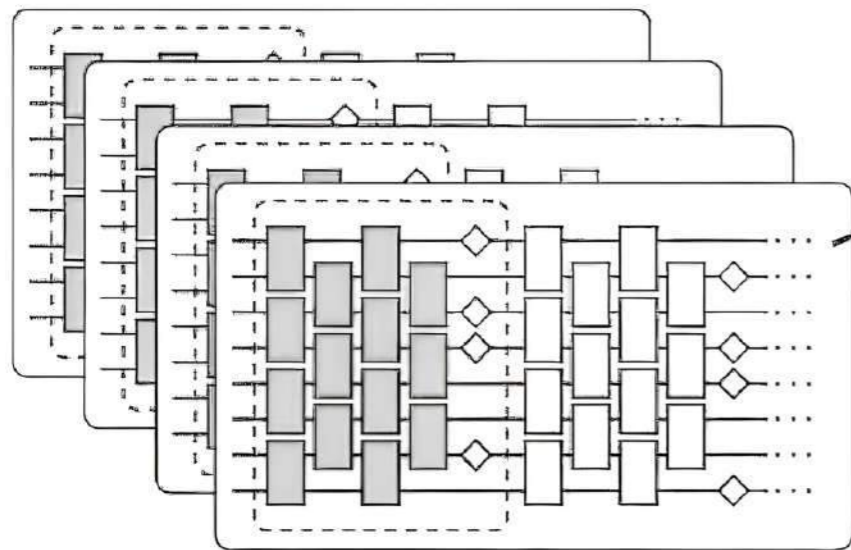
Experimental determination of entanglement entropy (EE) requires **quantum state tomography**



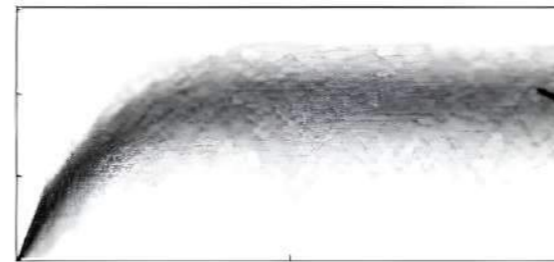
The standard way: **Pauli string operators** $\sigma_{i_1} \otimes \cdots \otimes \sigma_{i_N}$

$$4^N - 1 = (2^N + 1) (2^N - 1)$$

Highly resource- intensive quantum simulations

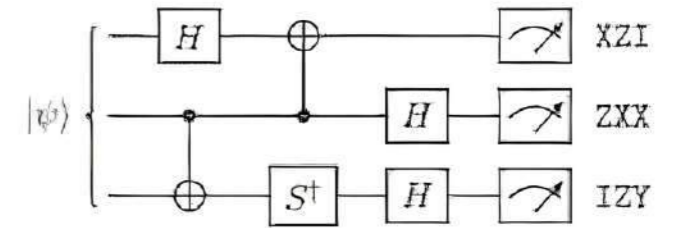


≥ 50-100 sampled random circuits



01010010101	ρ	S
01010000001	ρ	S
00000010000	ρ	S
⋮	⋮	⋮
01110010000	ρ	S

~ 2^{10} - 2^{16} trajectories / random circuit



~10 MUBs / trajectory
 ≥ 1000 shots / MUB

JM Koh et al., Nat. Phys. (2023)

Polynomial:

$$\mathcal{O}(L^\eta)$$

Exponential:

$$\mathcal{O}(e^{pLT})$$

Exponential:

$$\mathcal{O}(L^{\eta'} e^L)$$

Entanglement entropy & fluctuations

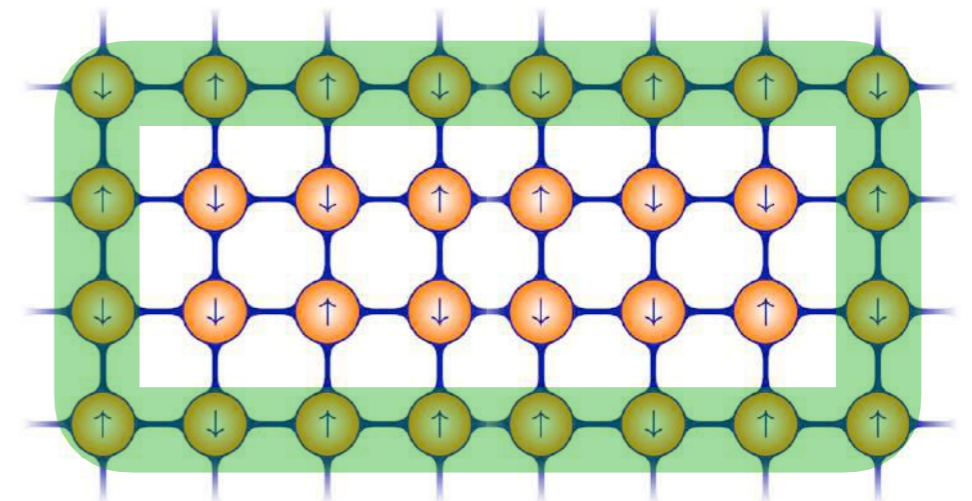
Microscopic distribution

$$\lambda_i \sim \frac{1}{2^{N_a}}$$

$$\mathcal{S}_2 = -\ln \sum_i \lambda_i^2 \sim N_a$$

$$\mathcal{S}_{vN} = -\sum_i \lambda_i \ln \lambda_i \sim N_a$$

N_a active spins (qubits)



Earlier works: Klich & Levitov *PRL* (2009)
Rachel et al., *PRL* (2012)
Song et al. *PRB* (2012)

Entanglement entropy & fluctuations

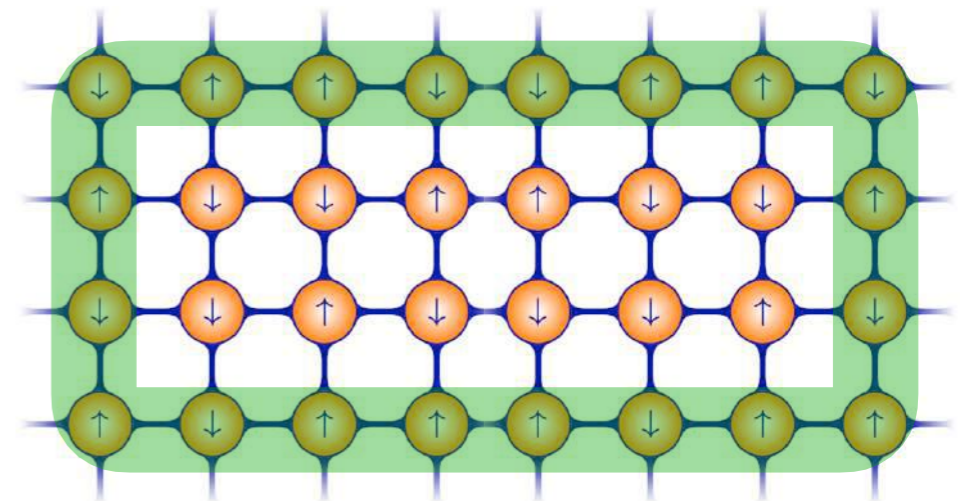
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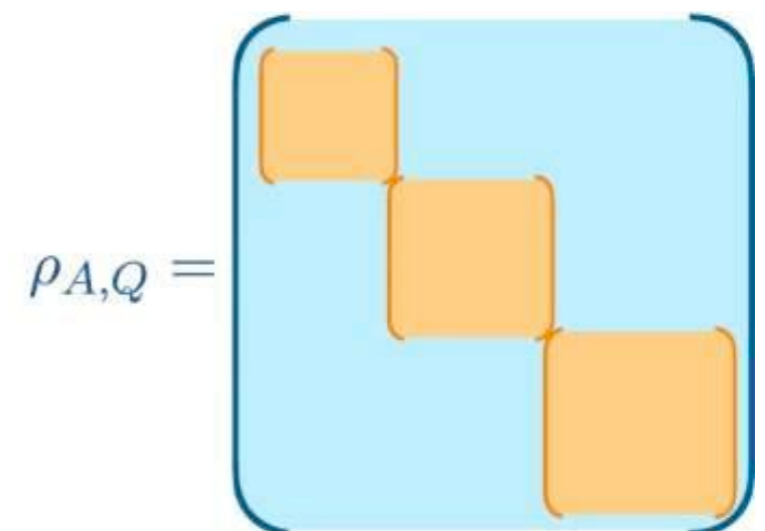
$$\mathcal{S}_{vN} = -\sum_i \lambda_i \ln \lambda_i \sim N_a$$

N_a active spins (qubits)



For a conserved extensive quantity

$$[Q, |\Psi\rangle\langle\Psi|] = 0, \quad Q = Q_A + Q_B \quad \Rightarrow \quad [Q_A, \rho_A] = 0$$



Goldstein & Sela, *PRL* (2018)

Ares, Murciano & Calabrese, *Nat. Commun.* (2023)

Entanglement entropy & fluctuations

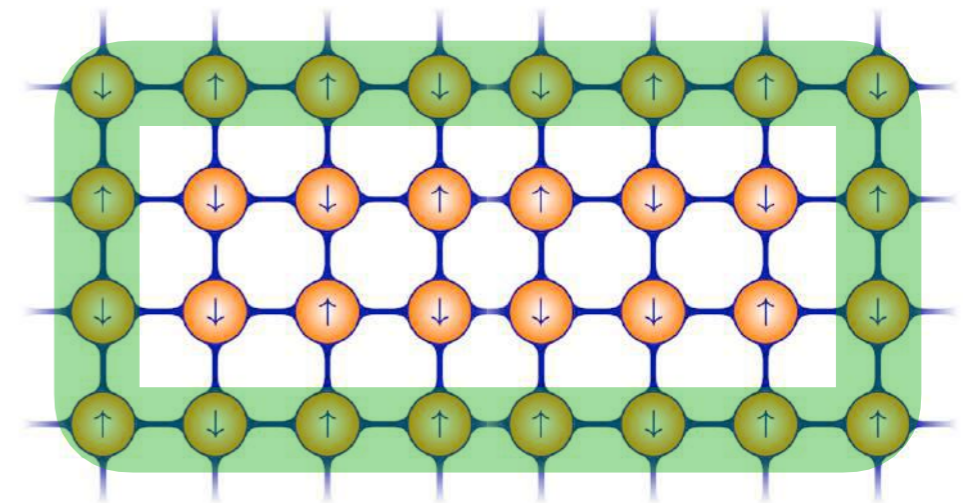
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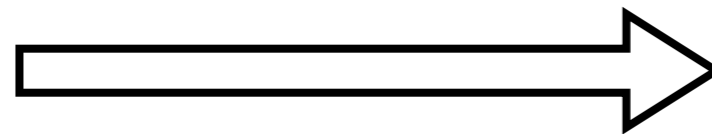
N_a active spins (qubits)



Coarse-grained distribution

$$\delta^2 Q_A = \frac{1}{2} \sum_{Q_i^A, Q_j^A} \lambda_{Q_i^A} \lambda_{Q_j^A} (Q_i^A - Q_j^A)^2$$

$$\lambda_{Q_i^A} \sim \binom{N_a}{Q_i^A} \frac{1}{2^{N_a}}$$



$$\delta^2 Q_A \sim N_a \sim \mathcal{S}$$

U(1)-symmetric monitored quantum dynamics

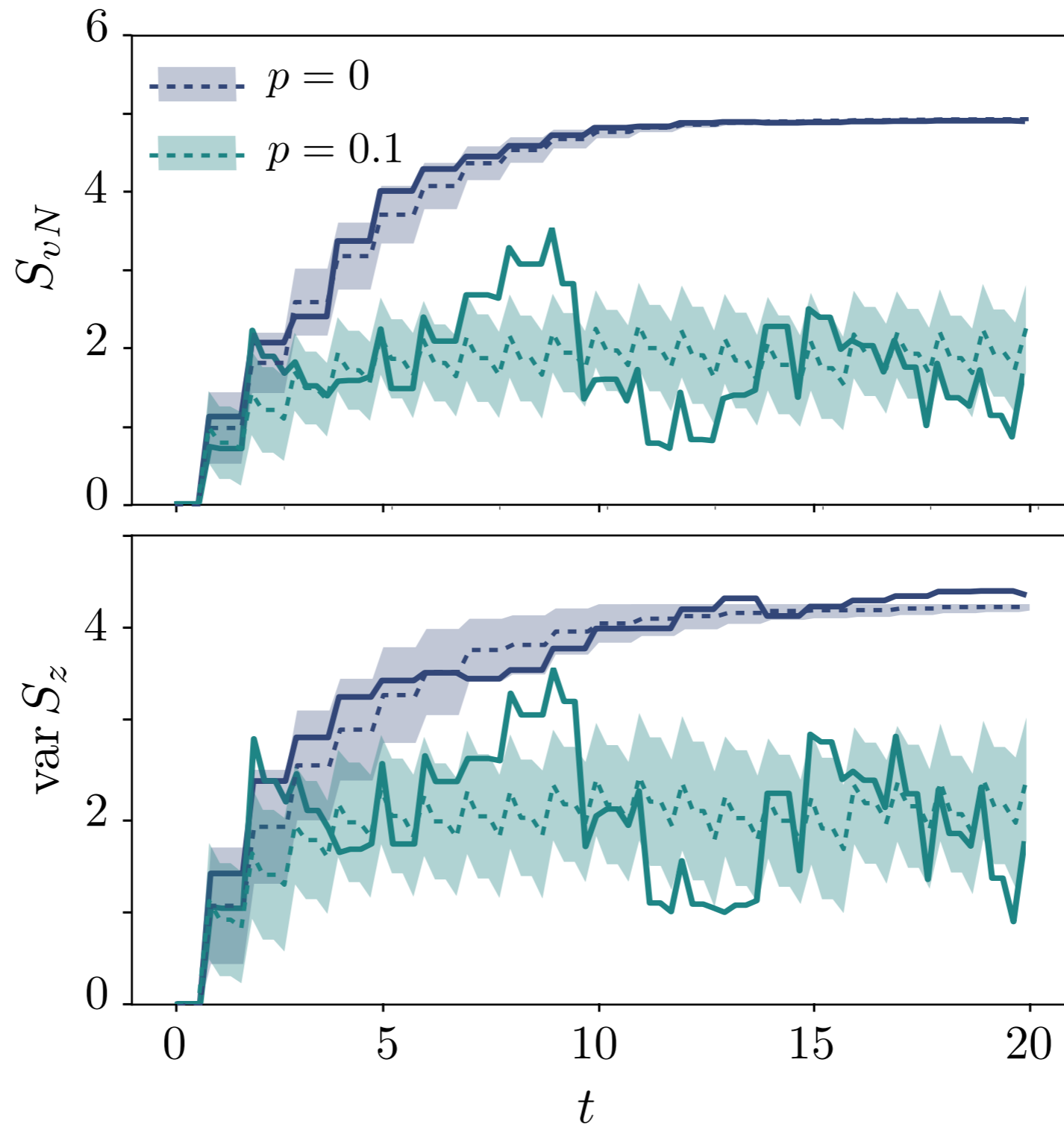
U(1) charge preserving two-qubit unitary

$$U^{j,j+1} = \begin{pmatrix} e^{i\phi_{00}} & & & \\ & U_{2 \times 2} & & \\ & & & e^{i\phi_{11}} \end{pmatrix} \quad \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$$

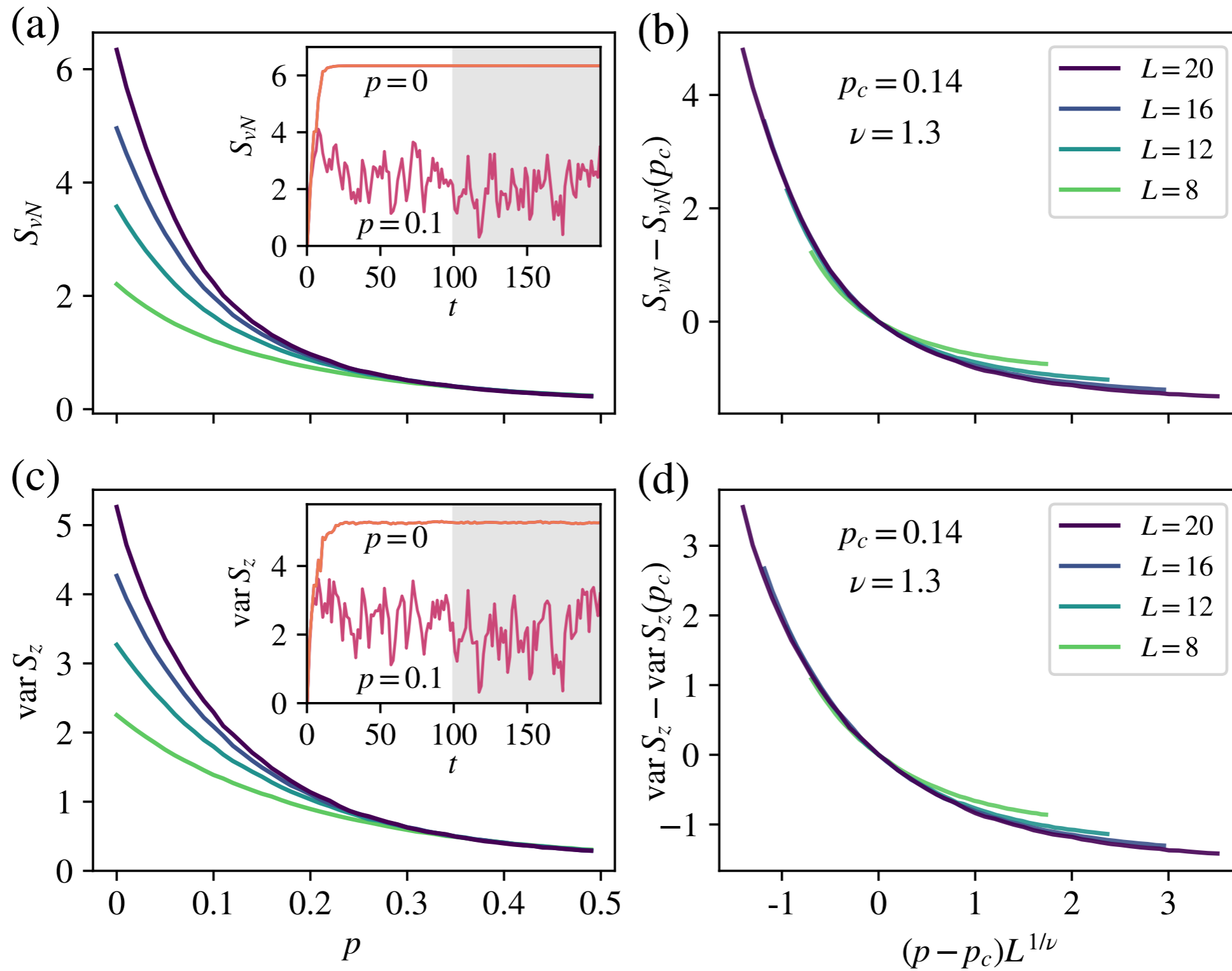
Measurements (in the charge basis) also preserve the total charge

Subsystem charge fluctuations to probe entanglement?

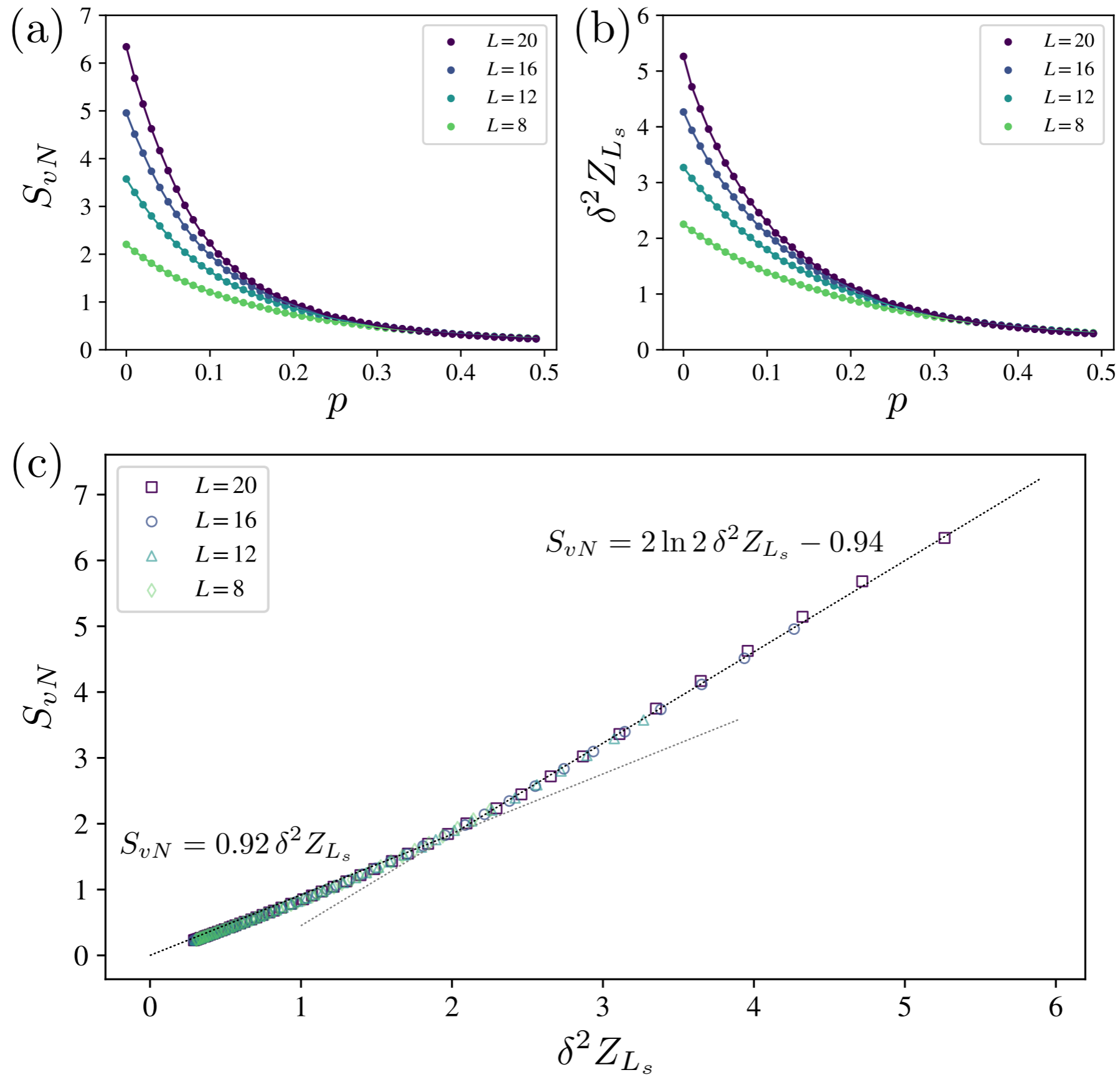
U(1)-symmetric monitored quantum dynamics



U(1)-symmetric monitored quantum dynamics



Full reconstruction of EE from charge variance



Adaptive steering to avoid postselection

w/o postselection we get highly mixed states

$$\overline{S}_{L_s}|_{\text{no PS}} = S_{L_s} \left(\sum_{\mathbf{m}} p_{\mathbf{m}} |\psi_{\mathbf{m}}\rangle \langle \psi_{\mathbf{m}}| \right) \equiv S_{L_s}(\bar{\rho}) \propto |L_s|$$

p independent

$$\overline{\delta^2 Z}_{L_s}|_{\text{no PS}} = \delta^2 Z_{L_s}(\bar{\rho}) \propto |L_s|$$

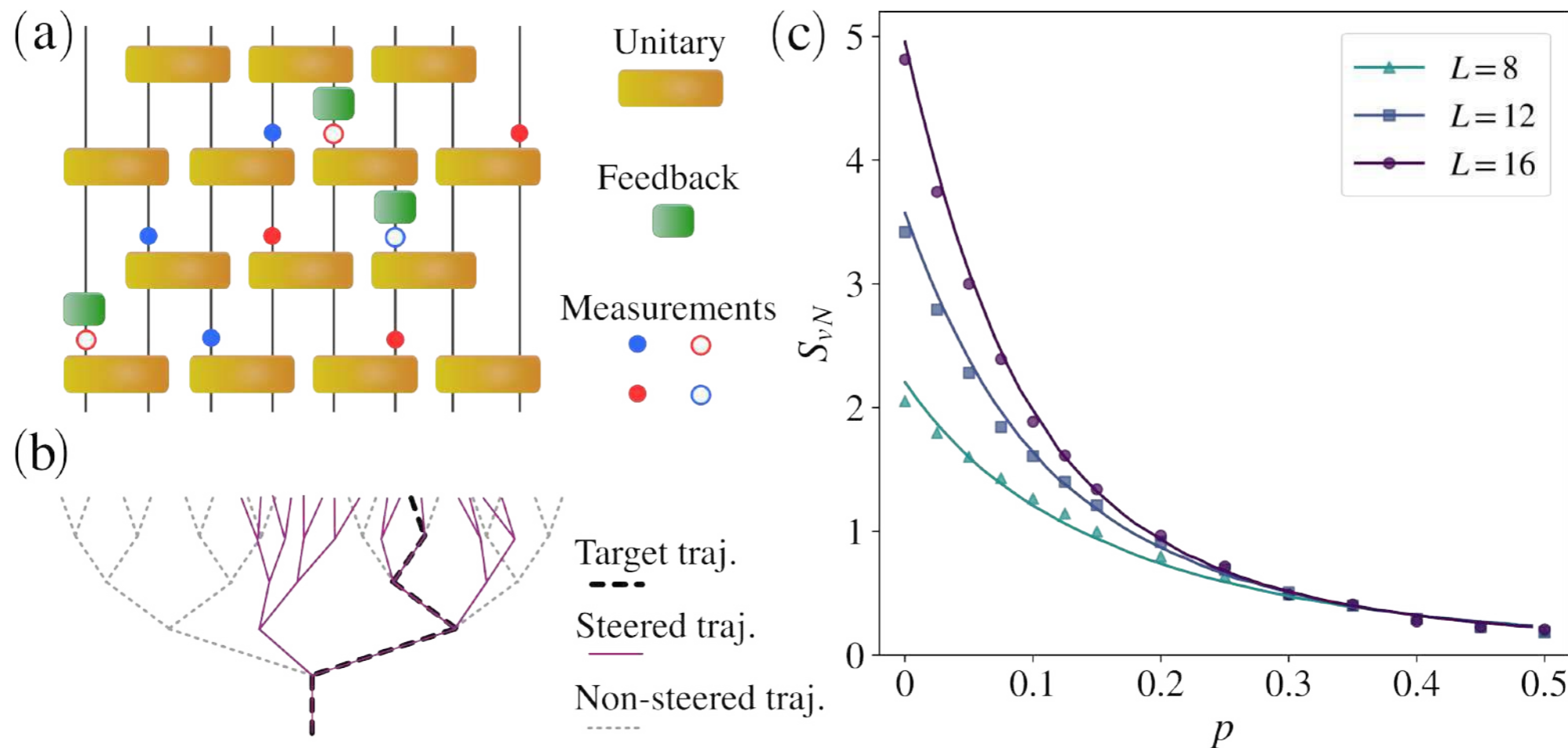
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p independent

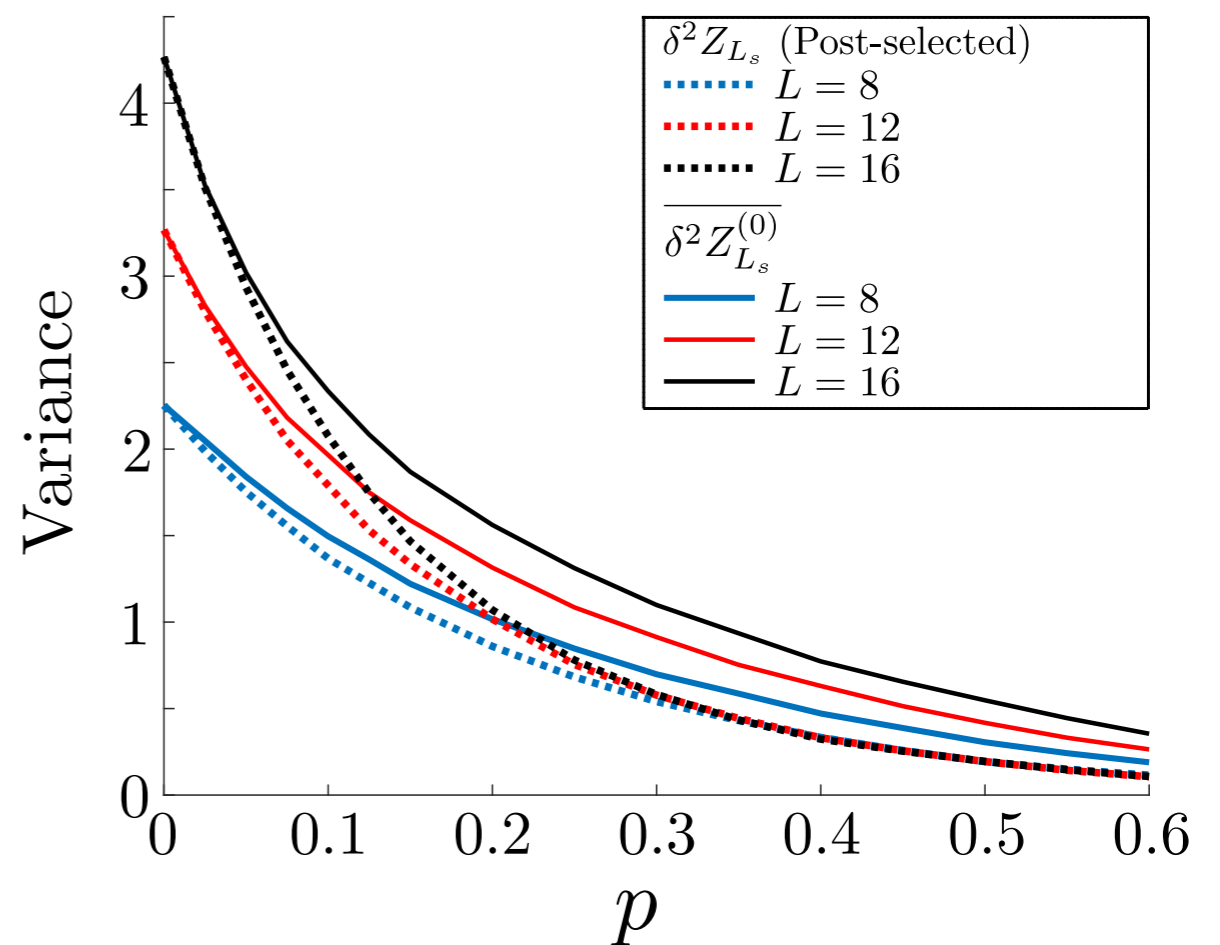
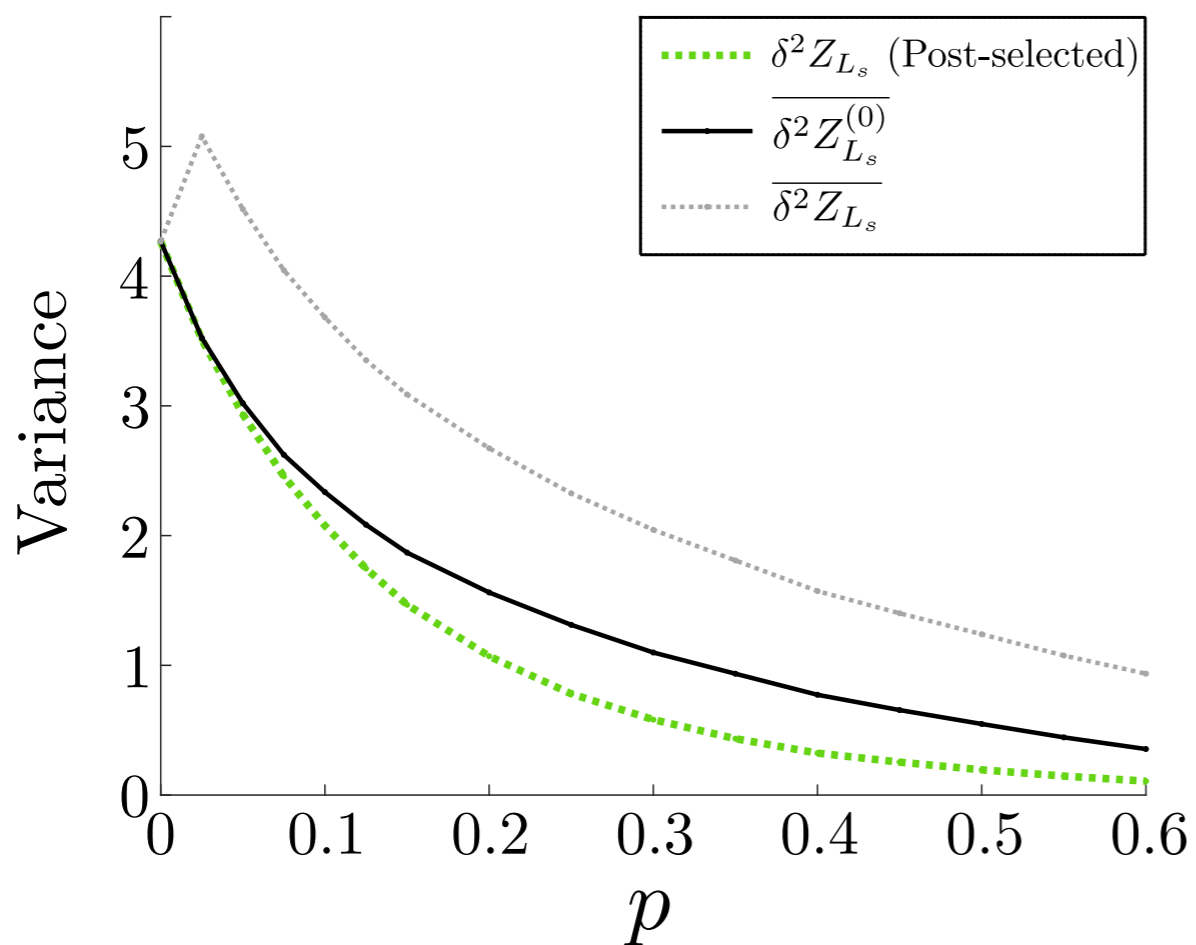
$$\overline{\delta^2 Z_{L_s}}|_{\text{no PS}} = \delta^2 Z_{L_s}(\bar{\rho}) \propto |L_s|$$



Applying corrective local feedbacks to steer towards the trajectory

$$\bar{\rho} \xrightarrow{\text{steering}} \rho_{\mathbf{m}} = \frac{1}{\mathcal{N}_{\mathbf{s}}} \sum_i |\Psi_{\mathbf{m}'_i \rightarrow \mathbf{m}}\rangle \langle \Psi_{\mathbf{m}'_i \rightarrow \mathbf{m}}| \quad \Rightarrow \quad \overline{\delta^2 Z_{L_s}}$$

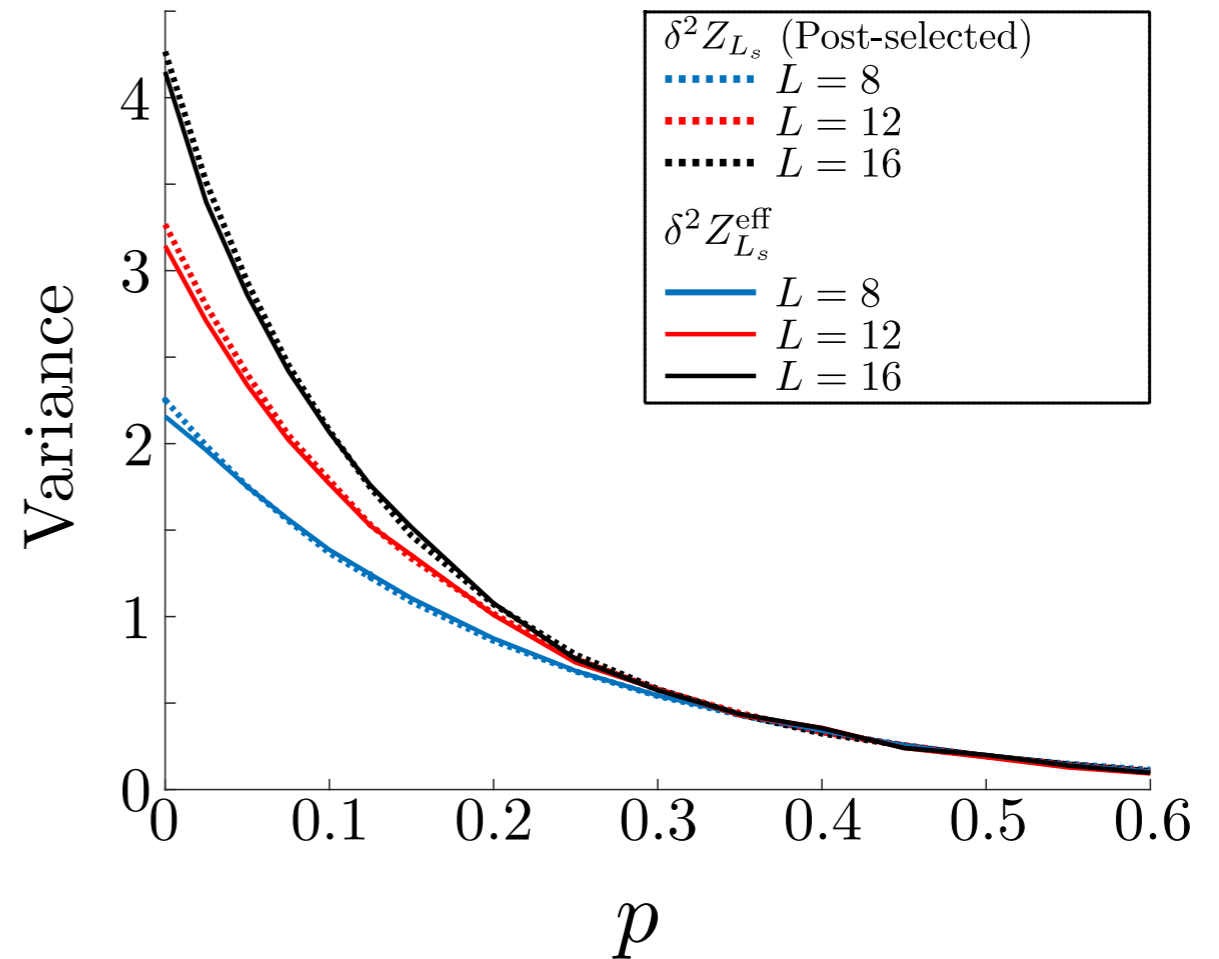
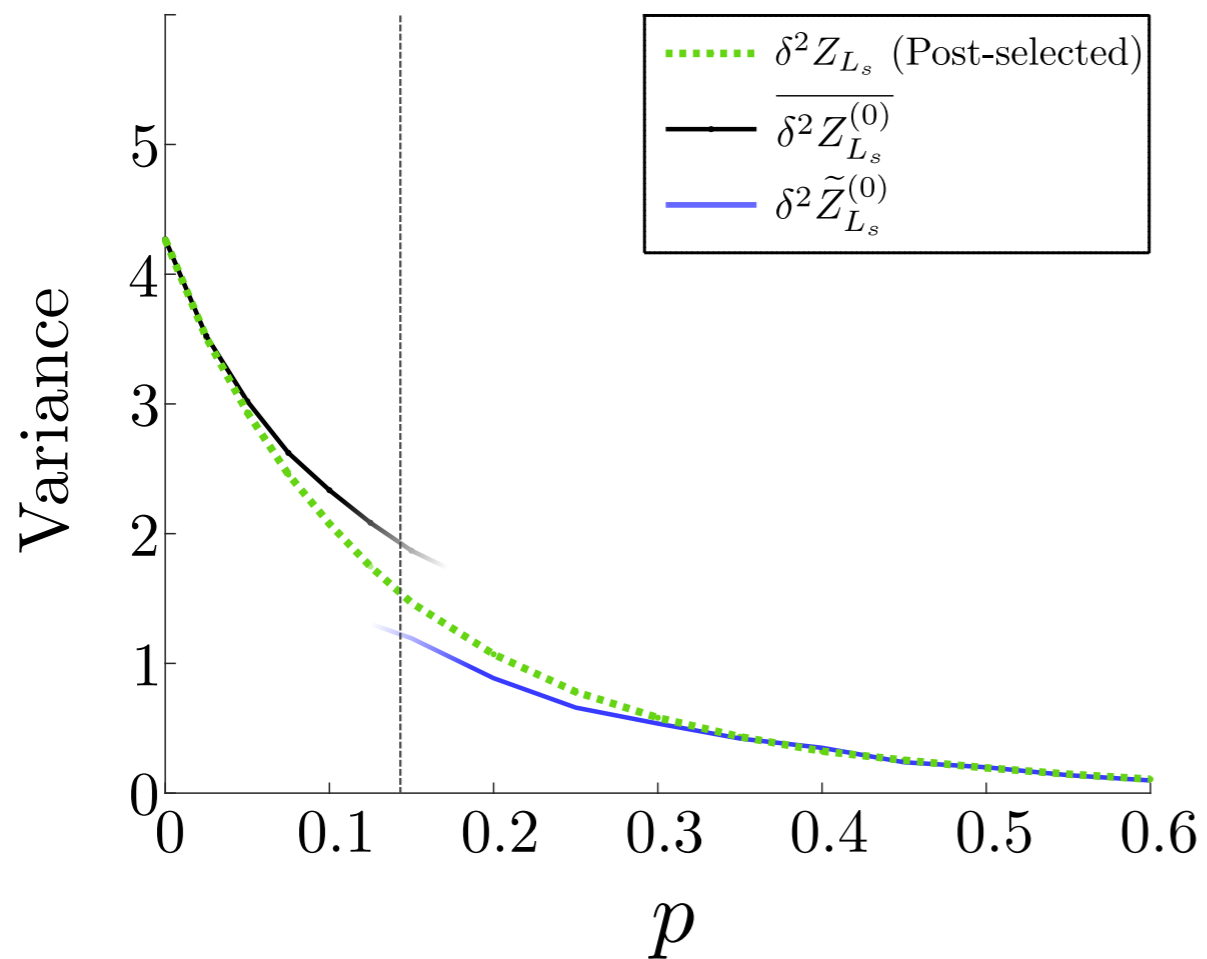
$$\tilde{\rho}_{\mathbf{m}}^{Z_L = \text{cte}} = \frac{1}{\mathcal{N}_{Z_L}} \sum_i |\Psi_{\mathbf{m}'_i \rightarrow \mathbf{m}}\rangle \langle \Psi_{\mathbf{m}'_i \rightarrow \mathbf{m}}| \quad \Rightarrow \quad \overline{\delta^2 Z_{L_s}^{(0)}}$$



Removing parasitic volume-law: $\delta^2 \tilde{Z}_{L_s}^{(0)} = \overline{\delta^2 Z_{L_s}^{(0)}} - c_V(p) [L_s - 2]$

Gluing area-and volume-law parts: $\delta^2 Z_{L_s}^{\text{eff}} = \overline{\delta^2 Z_{L_s}^{(0)}} - g(p) c_V(p) [L_s - 2]$

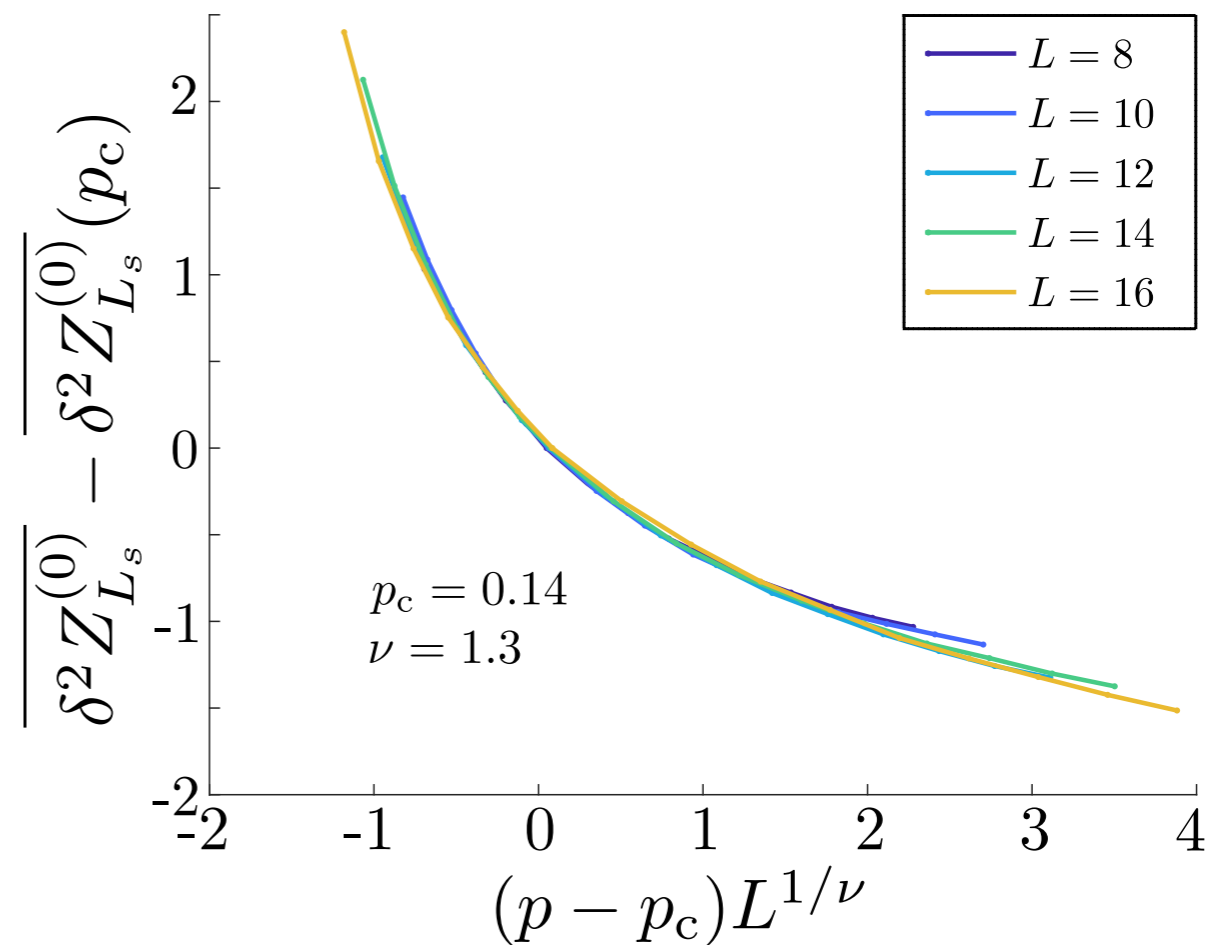
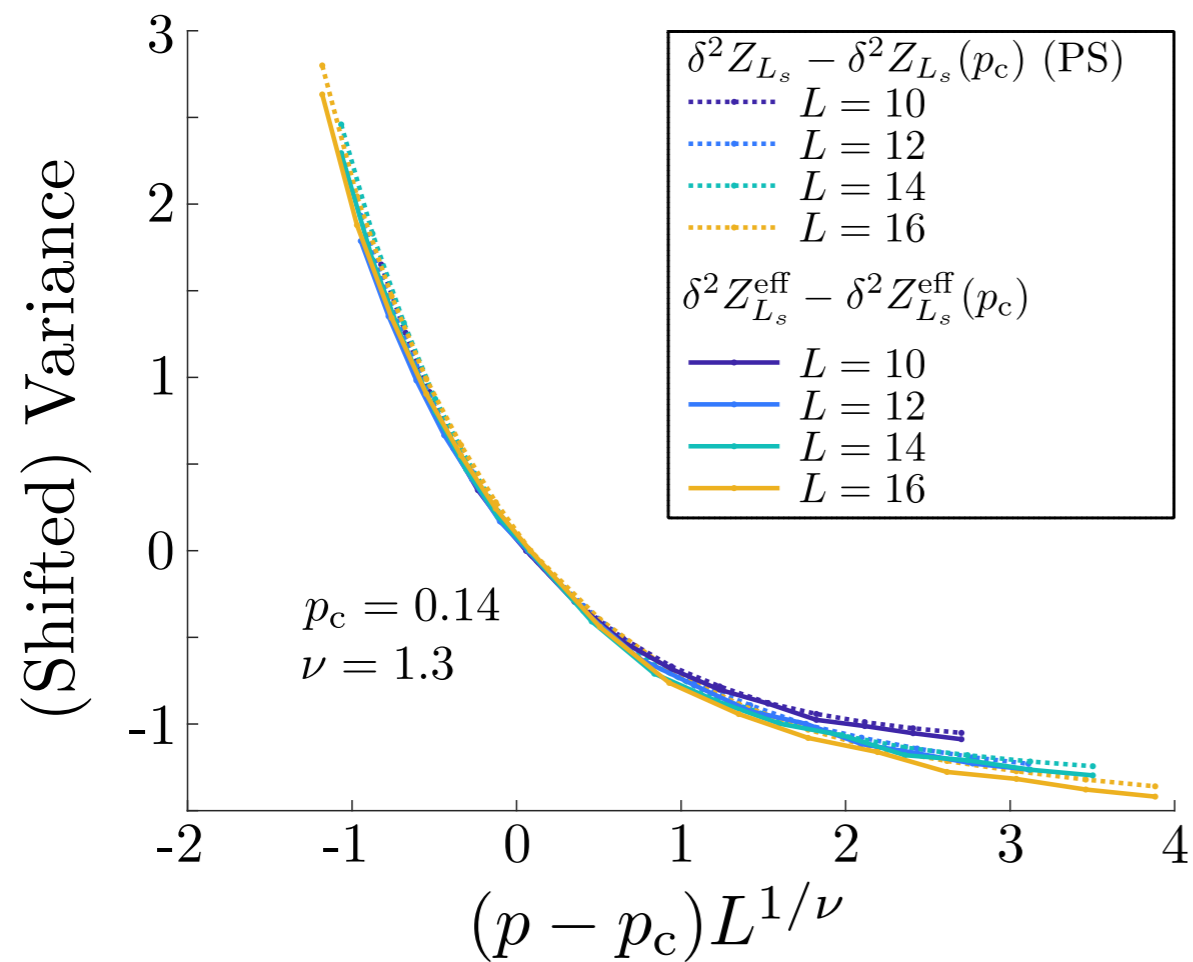
$$g(p) = G \left[(p - p_c) L^{1/\nu} \right] \quad G(x) = f_{\text{FD}}(-x)$$



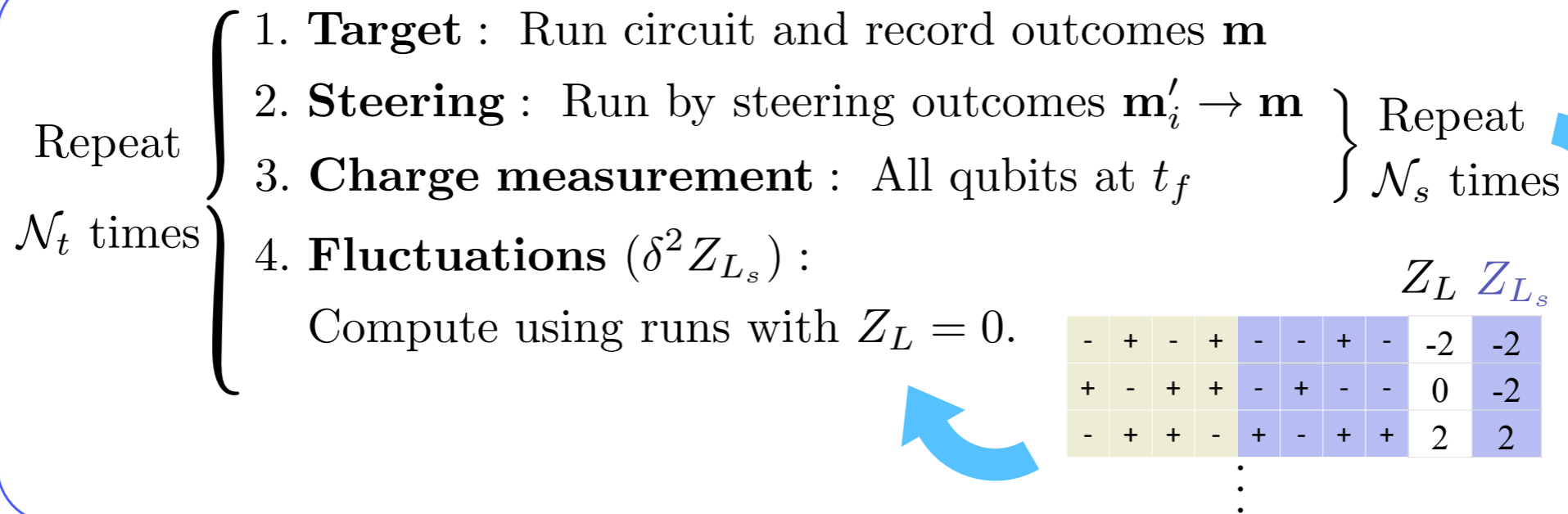
Single-parameter scaling ansatz:

$$F(p, L) - F(p_c, L) = \tilde{F} [(p - p_c)L^{1/\nu}]$$

Similar scaling for the corrected & uncorrected fluctuations (no theory input)



Data collection



Obtaining critical parameters

5. $\overline{\delta^2 Z_{L_s}^{(0)}}$: Averaged over many target trajectories results
6. **Scaling** : Find p_c, ν by collapsing $\overline{\delta^2 Z_{L_s}^{(0)}}$ for different length

Reconstructing (post-selected) quantities

7. **Parasitic term extraction** : $c_V(p) = (\overline{\delta^2 Z_{L_s=2k}^{(0)}} - \overline{\delta^2 Z_{L_s=2k-1}^{(0)}})/2$
8. **Parasitic term omission** :
 - (i) for $p > p_c$ subtract $c_V(p)$ from avg. fluctuations
 - (ii) Glue results for $p > p_c$ and $p < p_c$ by interpolated subtraction)

Quantum overhead required by the protocol

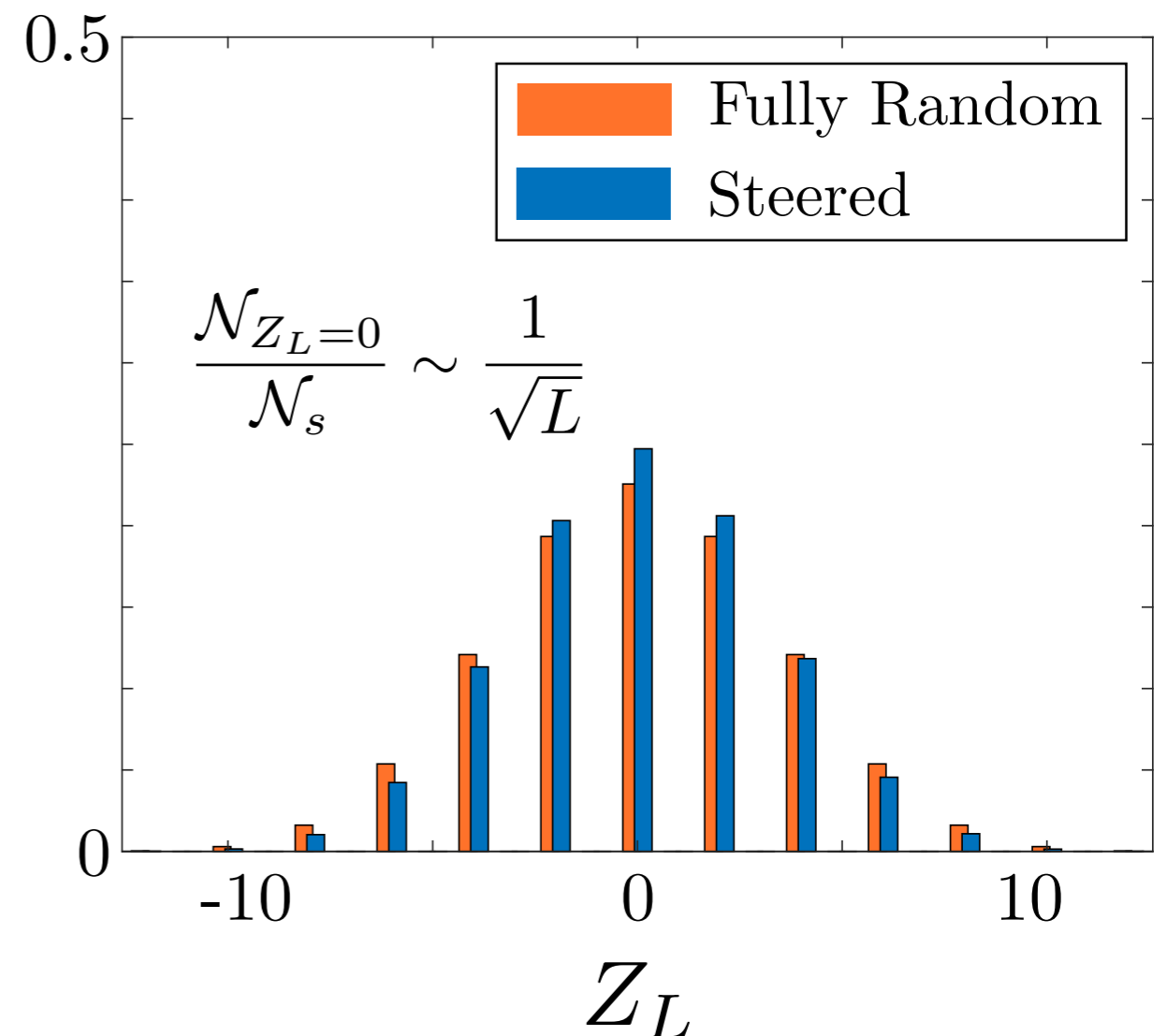
$$\delta^2 (\delta^2 Z) \sim \frac{2}{\mathcal{N}_{Z_L=0}} (\delta_\infty^2 Z)^2 \quad (\text{Sampling}) \text{ variance of variace}$$

$$\delta^2 (\delta^2 Z) < \epsilon$$

$$\mathcal{N}_{Z_L=0} > (\delta_\infty^2 Z)^2 (2/\epsilon)$$

Highly scalable overhead

$$\mathcal{N}_s \gtrsim \frac{\sqrt{L}}{\epsilon} (\delta_\infty^2 Z)^2 \sim \frac{L^{5/2}}{\epsilon}$$



Thank you!



Kim Pöyhönen
(Tampere)



Teemu Ojanen
(Tampere, HIP)



Moein N. Ivaki
(Aalto)



Research Council
of Finland

1. AGM, Pöyhönen, Ojanen, *PRX Quantum* (2023)
2. AGM, Pöyhönen, Ojanen, *PRL* (2023)
3. Pöyhönen, AGM, Ivaki, Ojanen, *arXiv:2406.19052*
4. Ivaki, Ojanen, AGM, (on the effect of noise) under preparation (2024)



JANE AND AATOS
ERKKO FOUNDATION

Summary

- Symmetry-enforced equivalence of entanglement and fluctuations
- Avoiding Full tomography can be by probing fluctuations
- Combining with steering eliminates the need for postselection
- Improved scalability and significantly reduced post-processing compared to other methods

- Stronger theoretical support is needed for our numerical findings?
- Postselection problem in other measurement-induced phase transitions?
- Effects of errors (depolarizing noise, decoherence)?