Fragility of quantum ergodicity

1. Porter-Thomas
2. Erlang
3. deep thermalization

} emergent Haar randomness

* Hilbert space delocalization & Porter-Thomas

$z_i \in \{0, 1\}$

IPR: $I_q = \sum_z |\langle z | \psi \rangle|^{2q}$

part. entropy $S_q = \frac{1}{1-q} \ln [I_q]$

Limits

* localized / product state

$I_q = 1, \quad S_q = 0$ area law

* Fully delocalized $|\langle z|\psi\rangle|^2 = 1/2^L$

$I_q = 2^L \times 2^{-Lq} = 2^{(1-q)L}$

$S_q = L \ln 2$ volume law

* Haar random state

$I_q = \frac{q!}{2^L(2^L+1)\dots(2^L+q-1)} \simeq 2^{L(1-q)} q!$

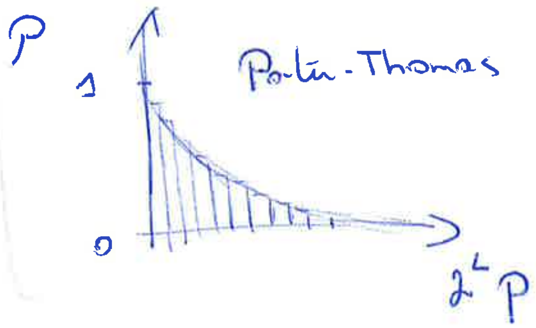
$S_q = L \ln(2) + \frac{\ln(q!)}{1-q}$ volume law

* Probability-of-probabilities

$p_z = |\langle z|\psi\rangle|^2 = |\langle z_1 z_2 \ldots z_L|\psi\rangle|^2$ bit-string prob.

$\rightsquigarrow P(p_z = |\langle z|\psi\rangle|^2)$

$\rightsquigarrow I_q = 2^L \int_0^1 dp\, P(p)\, p^q$ moments of prob. distribution



$\rightsquigarrow$ Haar-random: $P(p) = (N-1)(1-p)^{N-2}$ Beta

with $N = 2^L$

$\simeq N e^{-Np}$ [Porter-Thomas] Exponential

$$(N-1)\int_0^1 dp\,(1-p)^{N-2} p^q = \frac{q!\,(N-1)!}{(N+q-1)!}$$

Exponential: Gamma function

Sketch of proof

$$\int_{\phi \sim \text{Haar}} d\phi \, (|\phi\rangle\langle\phi|)^{\otimes q} = \sum_{\pi \in S_q} \frac{P(\pi)}{N(N+1)\dots(N+q-1)}$$

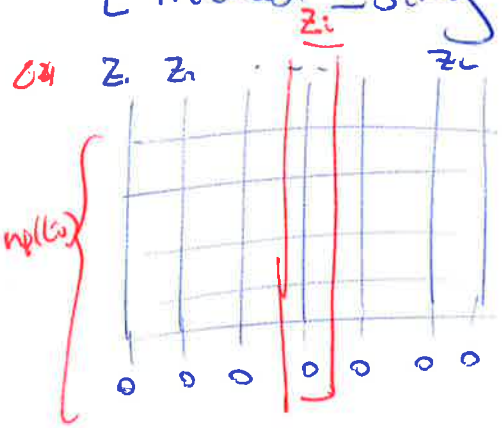
with $P(\pi)\, |i_1 i_2 \dots i_q\rangle = |i_{\pi(1)} i_{\pi(2)} \dots i_{\pi(q)}\rangle$

IPRS

$$\int d\phi \, (\langle z|\phi\rangle\langle\phi|z\rangle)^q = \sum_{\pi} \frac{\langle z\dots z|P(\pi)|z\dots z\rangle}{N(N+1)\dots(N+q-1)} = \frac{q!}{N(N+1)\dots(N+q-1)}$$

* Dual-unitary dynamics

[Kicked Ising Model]

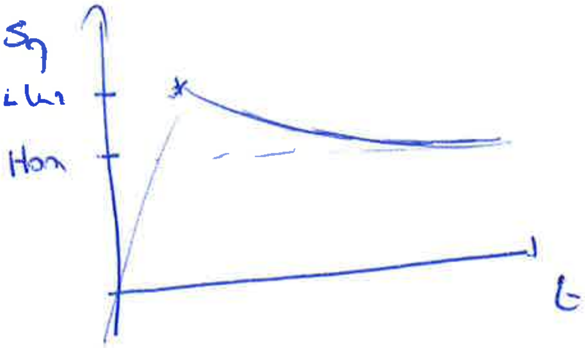


$$\rightsquigarrow \langle L| \tilde{U}(z_1)\tilde{U}(z_2)\dots\tilde{U}(z_L)|R\rangle \frac{1}{2^{L/2}}$$

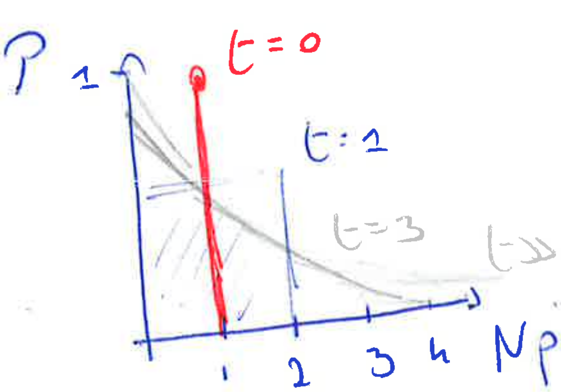
$$I_q = \sum_{z} |\langle L|\tilde{U}(z_1)\dots\tilde{U}(z_L)|R\rangle|^{2q} \frac{1}{2^{qL}}$$

$$= 2^{L(1-q)} \, \mathbb{E}\left[|\langle L|U|R\rangle|^{2q}\right]$$

$$\rightsquigarrow S_q = L\ln(2) + \frac{1}{1-q} \ln\left[\frac{q!\, 2^{qt}}{2^t(2^t+1)\dots(2^t+q-1)}\right]$$



$$\Rightarrow P(p;t) \sim \left(1 - \frac{Np}{2^t}\right)^{2^t - 2} \Theta(2^t - Np)$$



→ Emergent p_T

→ threshold independent of system size

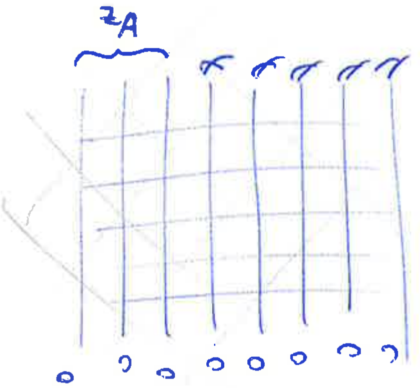
→ local vs global randomness

* Erlang distribution

⇝ partial access to bit-strings?

$$p(z_2, \dots z_L) = \sum_{z_1} p(z_1, z_2 \dots z_L)$$

(no access = RDM)



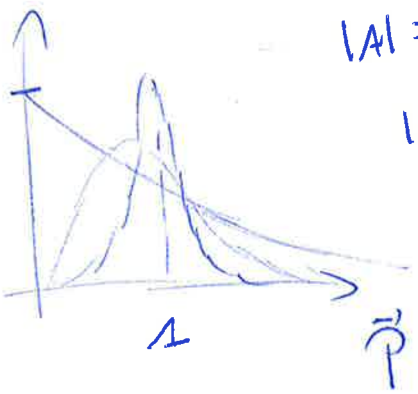
⇝ Haar-random state

$$P_{Erlang}(\tilde{p}, N_A) = \frac{N_A^{N_A}}{(N_A - 1)!} \exp(-N_A \tilde{p})\, \tilde{p}^{N_A - 1}$$

with $\tilde{p} = N/N_A \, \tilde{p}$

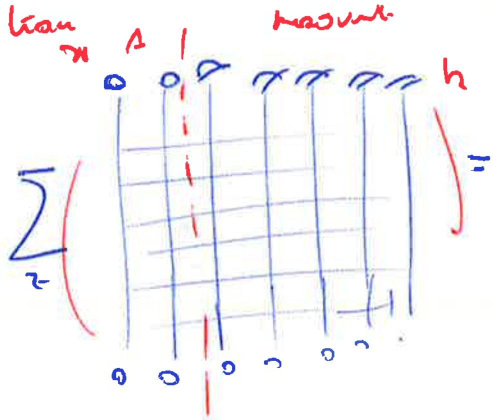
$|A| = ?,\ N_A = 1$: Porter-Thomas

$|A| \gg 1,\ N_A \gg 1$: Gaussian distribution with $\mu = 1$ and $\sigma^2 = 1/N_A$

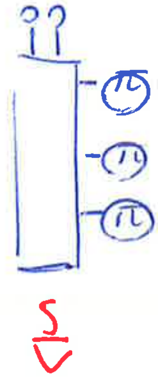


Interpretation: N_A i.i.d. exponential PT distributions lead to Erlang

⇝ Dual-unitary dynamics



$= \sum_{\tau}^{t} = \dfrac{1}{N_t(N_t+1)\cdots(N_t+q-1)} \sum_{\pi}$



$\rho = \sqcap\sqcap\sqcap\cdots\sqcap$ use that $\circ\!-\!(\pi) = 2^{\#\text{cycles in } \pi}$

$(\pi) = \ldots$ $\pi(1)\,\pi(2)\ldots$

$$= \frac{N_A(N_A+1)\cdots(N_A+q-1)\,\cancel{2^{t}}\,N_t^{q}}{N_t(N_t+1)\cdots(N_t+q-1)\,N_A^{q}}$$

$t \gg |A|$: $\dfrac{N_A(N_A+1)\cdots(N_A+q-1)}{N_A^{q}}$

$\left[{q \atop p} \right]$ - Stirling numbers first kind

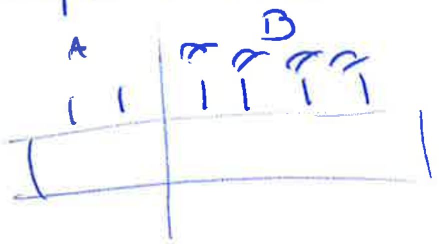
- # permutations with p cycles of q elements

$$\sum_{p=1}^{q} \left[{q \atop p} \right] x^{p} = x(x+1)\cdots(x+q-1)$$

$|A| = t$ ⇝ 1, delta function

$|A| > t$ ⇝ stays delta function

Deep thermalization



$\{|\psi_A(z)\rangle, p(z)\}$

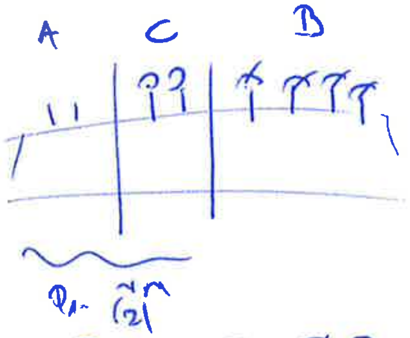
Late times $\{|\phi_A\rangle \;\; \phi_A \sim \text{Haar}\}$

$$\rho_A = \sum p(z)|\psi_A(z)\rangle\langle\psi_A(z)| \rightsquigarrow \mathbb{1}_A/N_A$$

$$\rho_A^{(2)} = \sum p(z)\left(|\psi_A(z)\rangle\langle\psi_A(z)|\right)^{\otimes 2} \rightsquigarrow \frac{1}{N_A(N_A+1)}\left[\mathbb{1}_{A\times A} + S_{A\times A}\right]$$

$$\mathbb{1} + S$$

partial access?



$\rightsquigarrow \rho_A = \rho_A \otimes \rho_A$

* limit $|C| > 2t$

$$\mathrm{Tr}_C\left[|\psi_A(z)\rangle\langle\psi_A(z)|\right] = \rho_A, \; \forall z$$

* consider $|C| < 2t$, deep thermaliz.

$$|\Phi_{A+C}\rangle \sim \Phi \;\text{Haar}$$

ρ_A — $|\Phi_{A+C}$

$$\rho_A^{(2)} \sim \int d\Phi_{A+C}\, \mathrm{Tr}_C\left[|\Phi\rangle\langle\Phi|_{A+C}\right] \otimes \mathrm{Tr}_C\left[|\Phi\rangle\langle\Phi|_{A+C}\right]$$

$$= \frac{1}{N_{A+C}(N_{A+C})}\left[\; \mathbb{1}_A \;\; \mathbb{1}_C - \mathbb{1}_C \;+\; S_A \;\; S_C - \mathbb{1}_C \;\right]$$

$\mathbb{1}_C - \mathbb{1}_C = N_C^2$, $\quad S_C - \mathbb{1}_C = N_C$

$$\mathfrak{Z}_A^{(2)} = \frac{1}{N_A N_c (N_A N_c + 1)} \left[N_c^2 \, \mathbb{1}_{A\times A} + N_c \, S_{A\times A} \right]$$

$$\lim_{N_c \gg 1} \mathfrak{Z}_A^{(2)} = \frac{\mathbb{1}_{A\times A}}{N_A^2}$$

as exponentially fast

[same holds for higher moments]

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