Geometric constructions of generalized dual-unitary circuits from biunitarity

DQI Group Meeting, 20.01.2025

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Outline

- Generalized dual unitarity
- Biunitary connections
- Solvable dynamics from biunitary connections
- Multilayer circuits

Motivation

- Understand common properties of **generic** physical systems
  → Thermalization, scrambling, diffusive transport, chaos,…

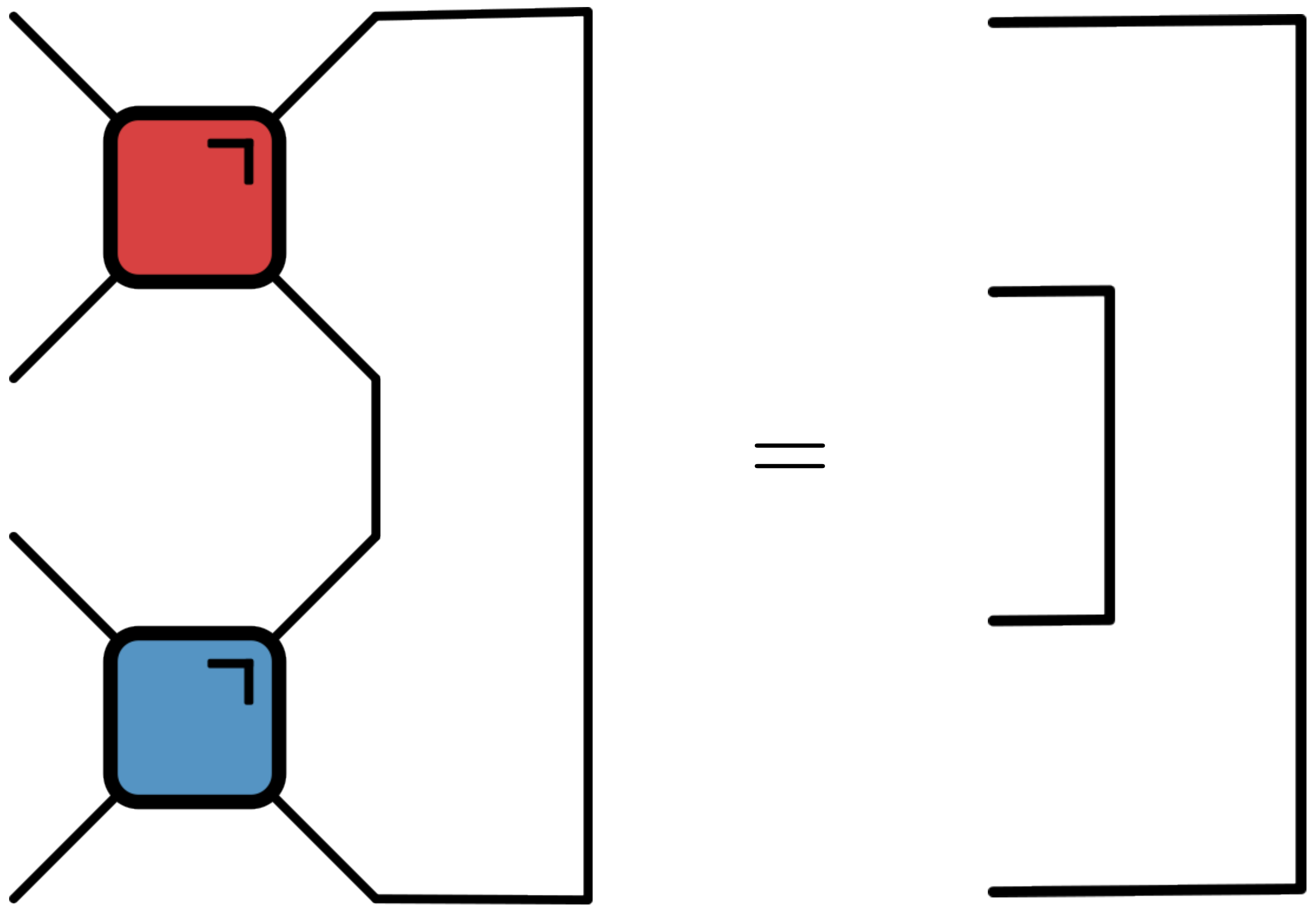
D'Alessio et al. (2016)

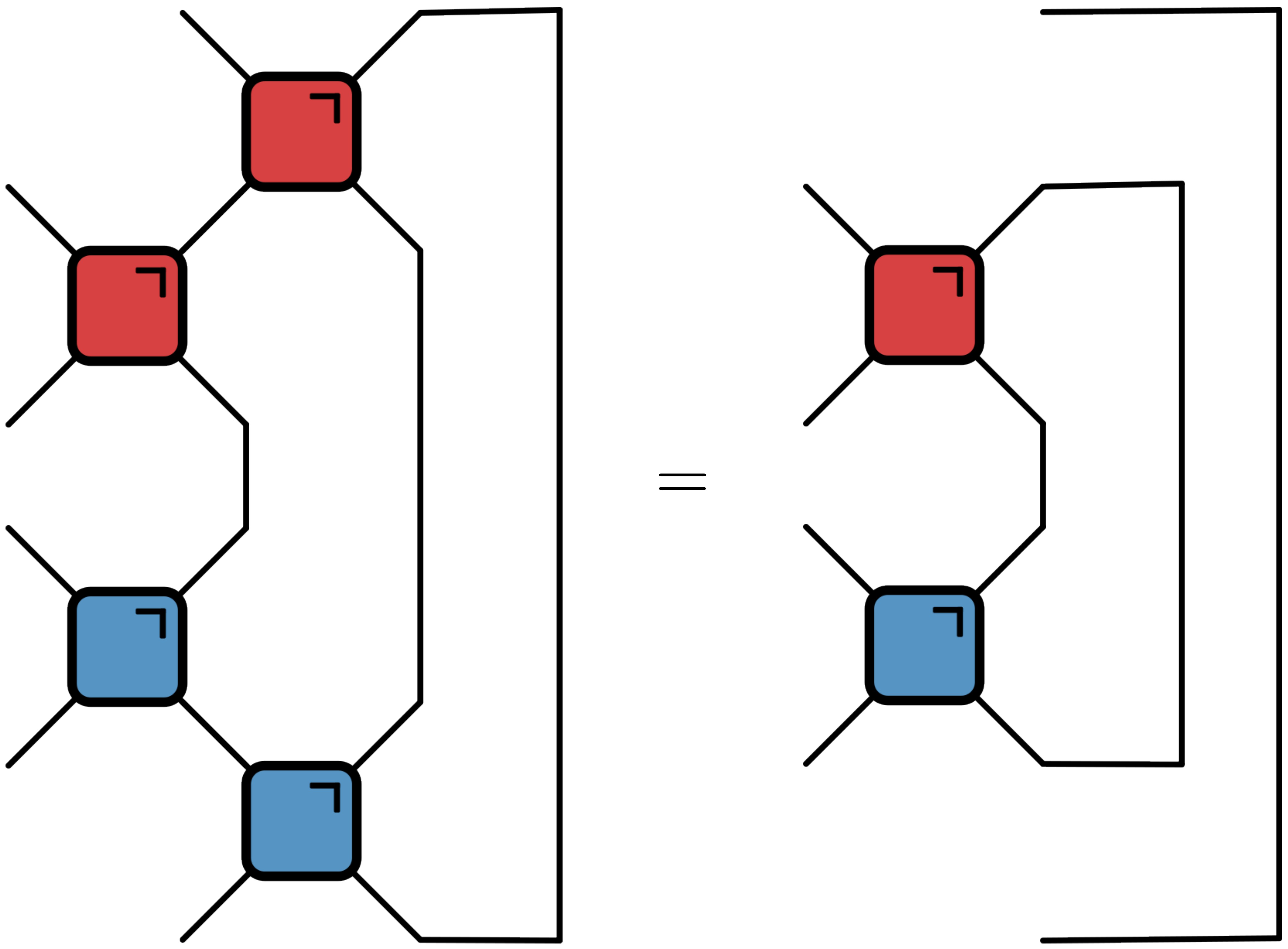
- Find ways how ergodicity can be broken
  → Integrability, disorder, kinetic constraints,…

Generalized dual unitarity

Hierarchically generalized dual unitarity

Dual unitary → DU2

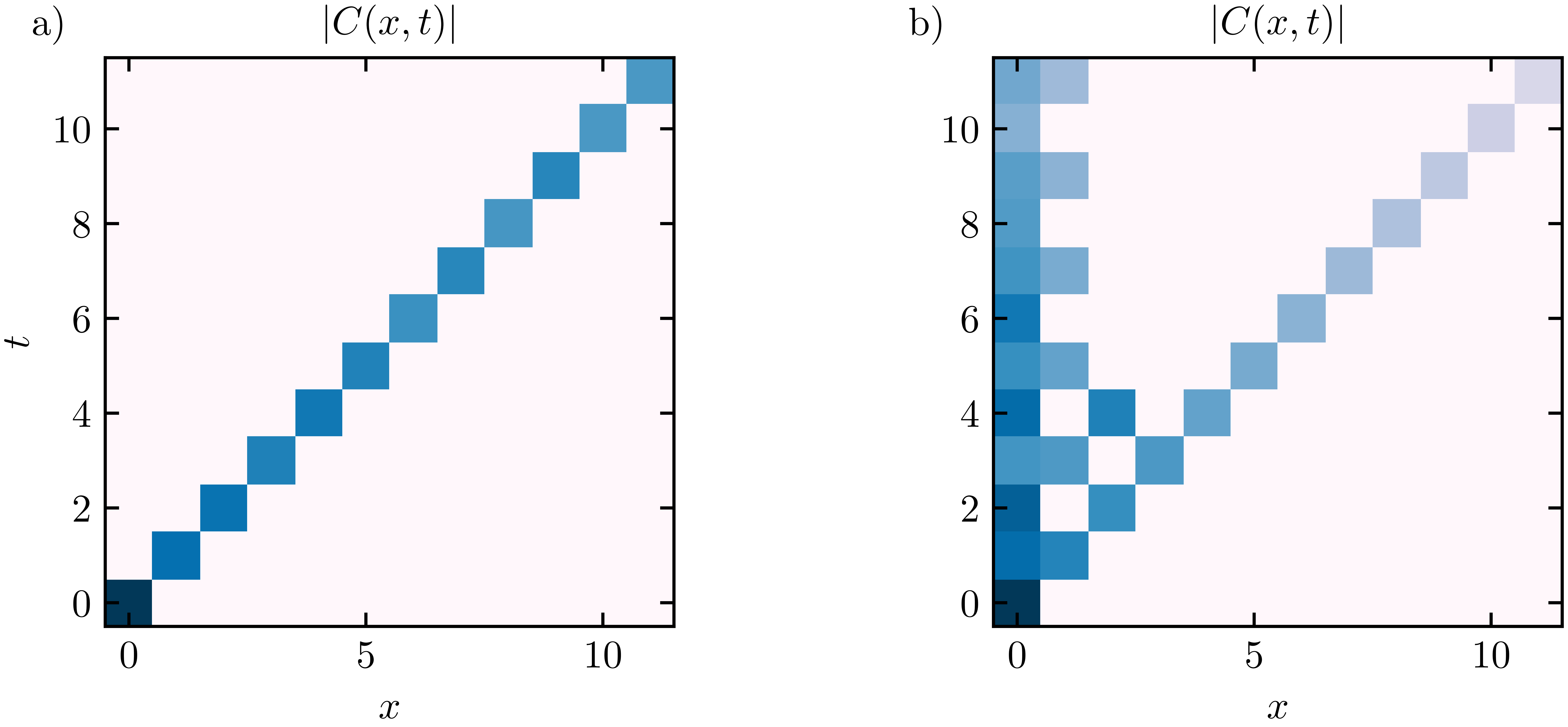




Yu, Wang, Kos (2023)

Correlations

Dual unitary → DU2



Schmidt decomposition

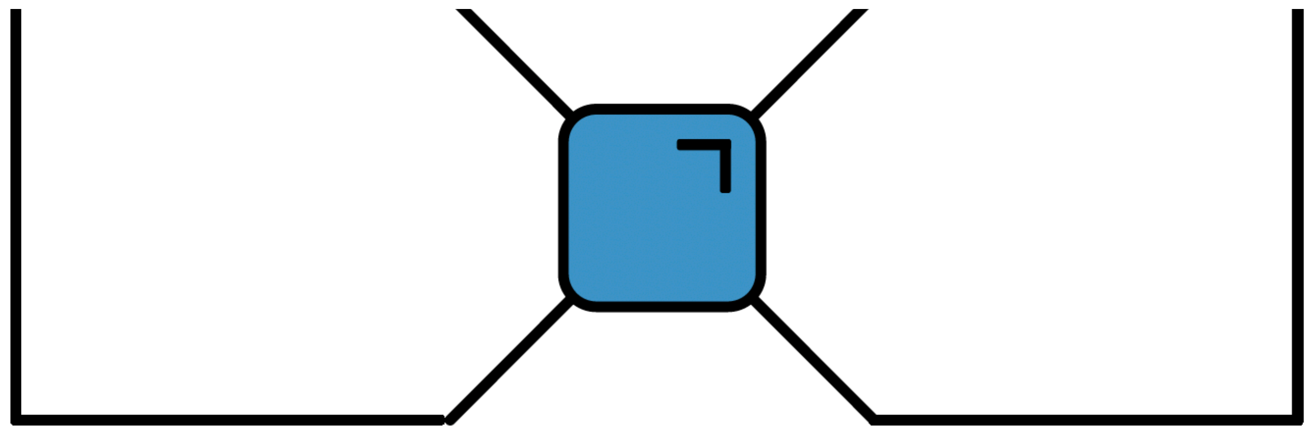
$$U = \sum_{i=1}^{q^2} \lambda_i X_i \otimes Y_i, \; \mathrm{tr}(X_i^\dagger X_j) = \mathrm{tr}(Y_i^\dagger Y_j) = \delta_{ij}$$

Normalization

$$\sum_i \lambda_i^2 = q^2$$

Operator entanglement $$E(U) = 1 - \frac{1}{q^4} \sum_{i=1}^{q^2} \lambda_i^4$$

Dual unitarity iff operator entanglement is maximal $$E(U) = 1 - \frac{1}{q^2}$$



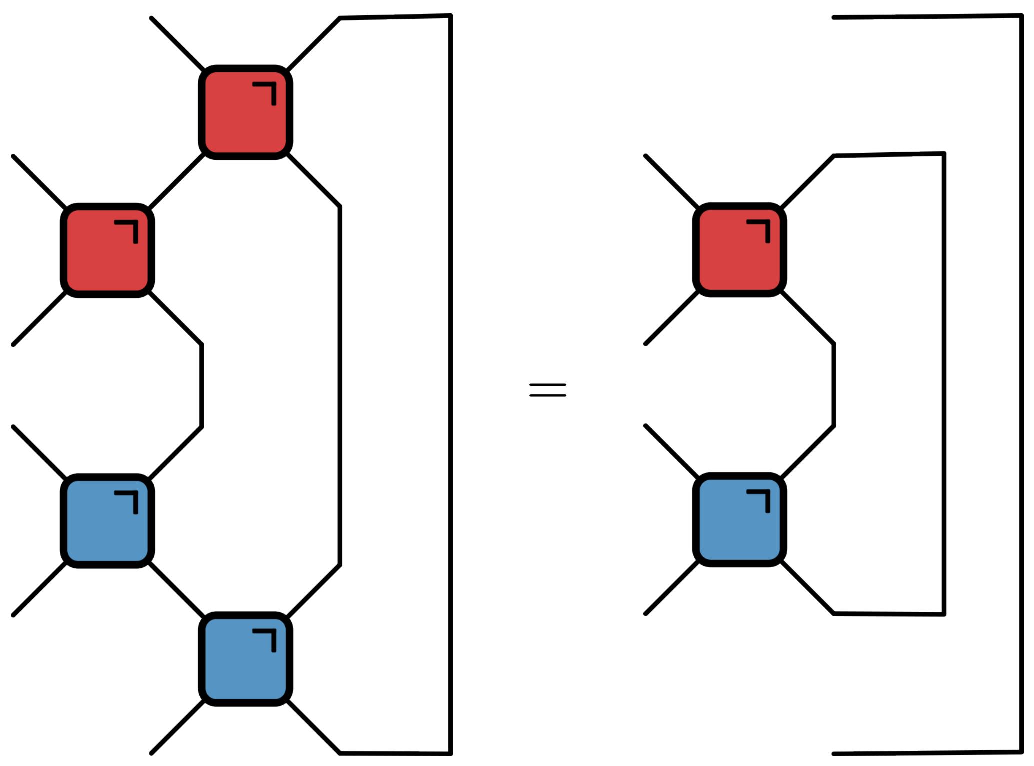
Constructing DU2 Gates

- Dual unitarity = maximal operator entanglement
  → invariant under local transformations

- DU2 breaks local invariance

- Necessary condition for DU2: flat Schmidt spectrum
  → characterize by Schmidt rank $\mathscr{R}$

Foligno, Kos, Bertini (2024)



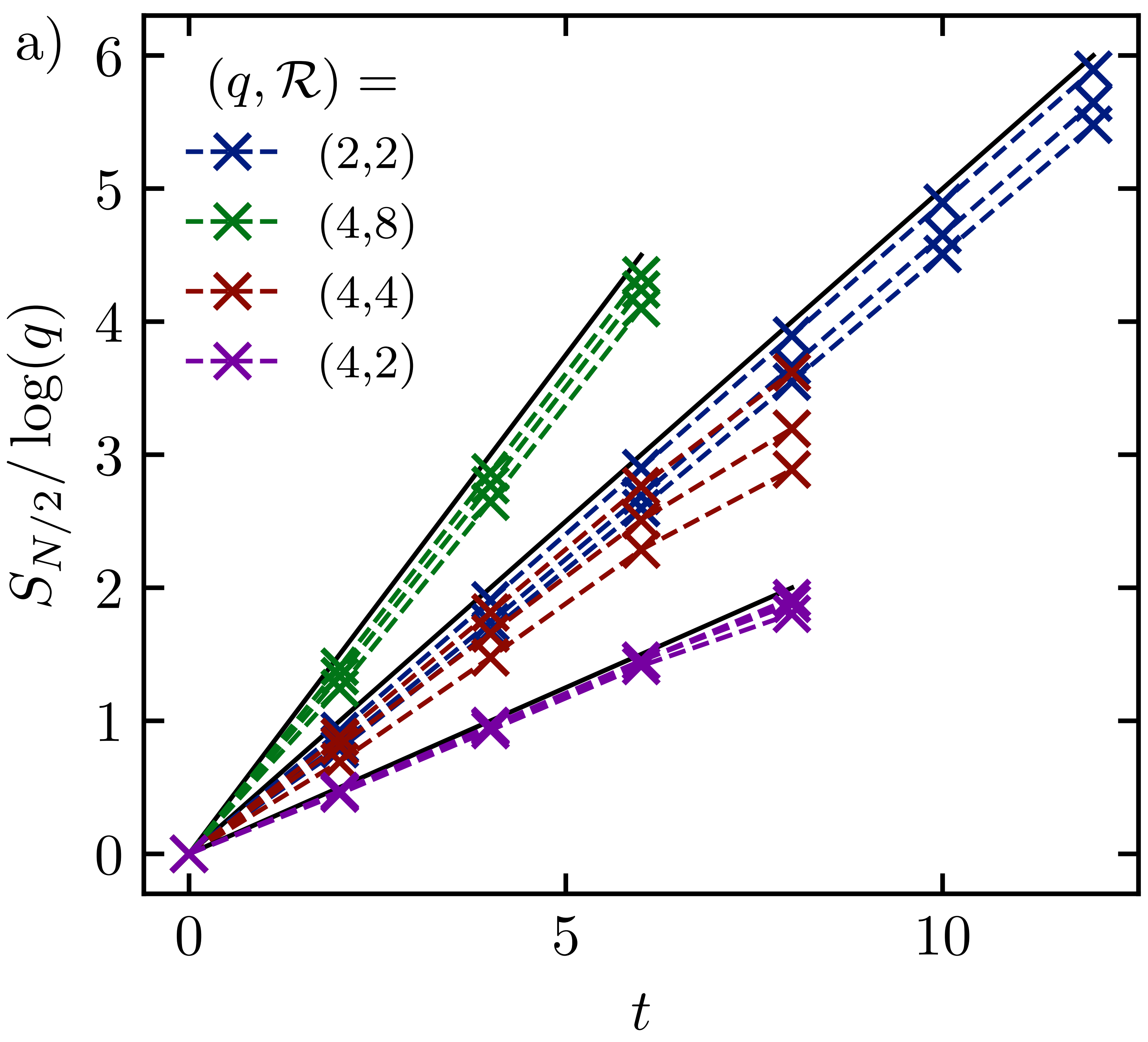
- Important example: CNOT

- Complete characterization only for qubits $q = 2$

Entanglement Growth

- Can be characterized exactly
- Entanglement velocity quantized

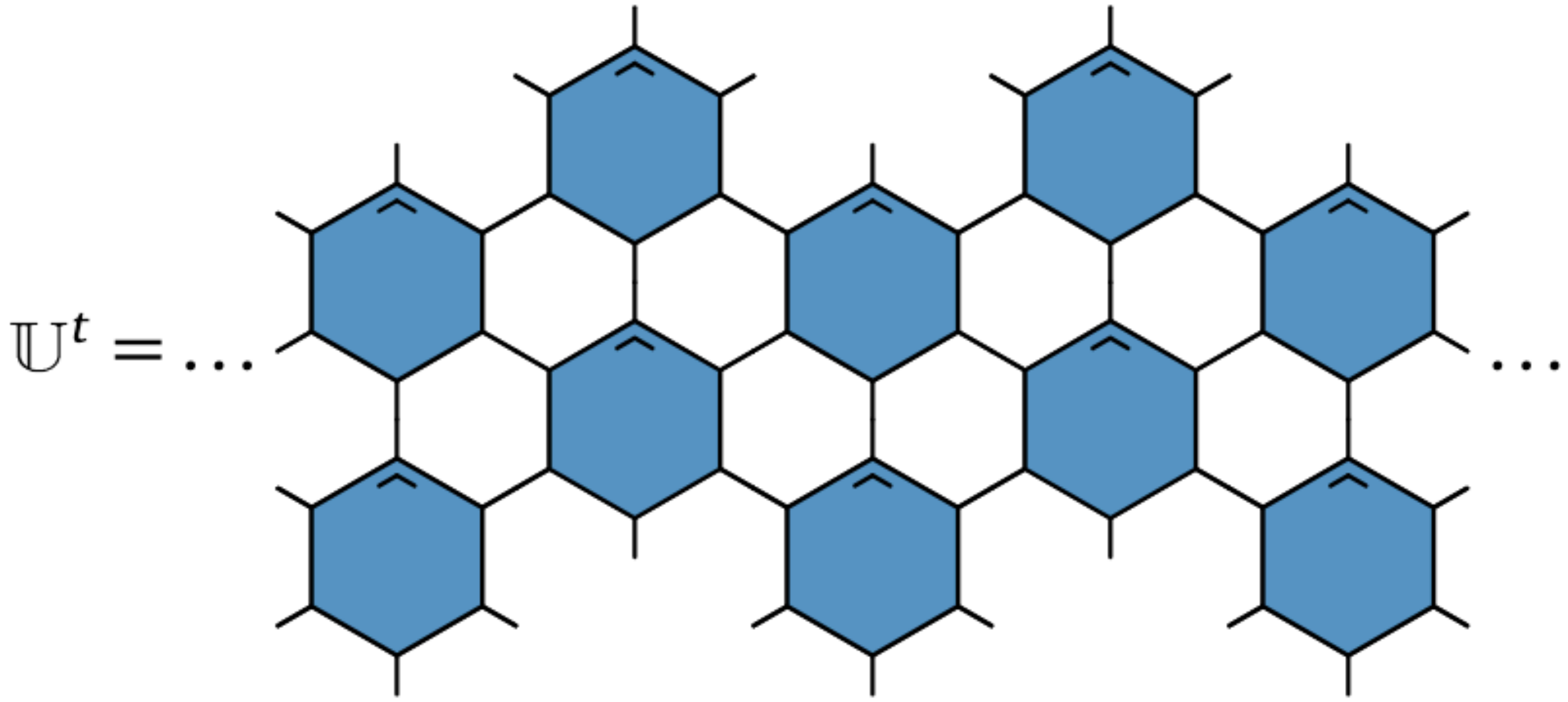
$$v_E = \frac{\log \mathcal{R}}{\log q^2}$$



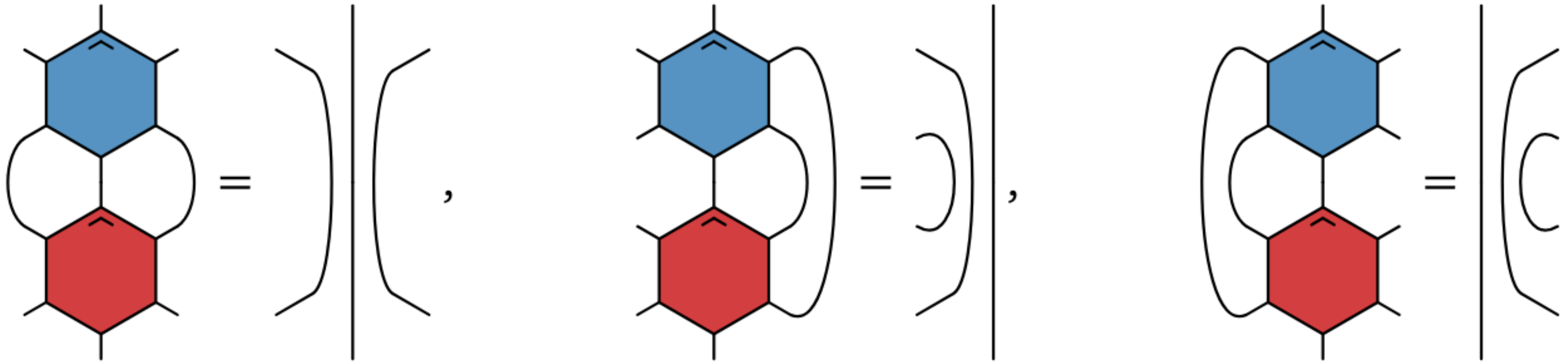
Rampp, Rather, Claeys (2024)

Triunitarity

- Circuit composed of three-site gates



- Unitary in three directions



- Dynamics similar to DU2 circuits
- Entanglement velocity fixed $v_E = 1/2$

Jonay, Khemani, Ippoliti (2021)

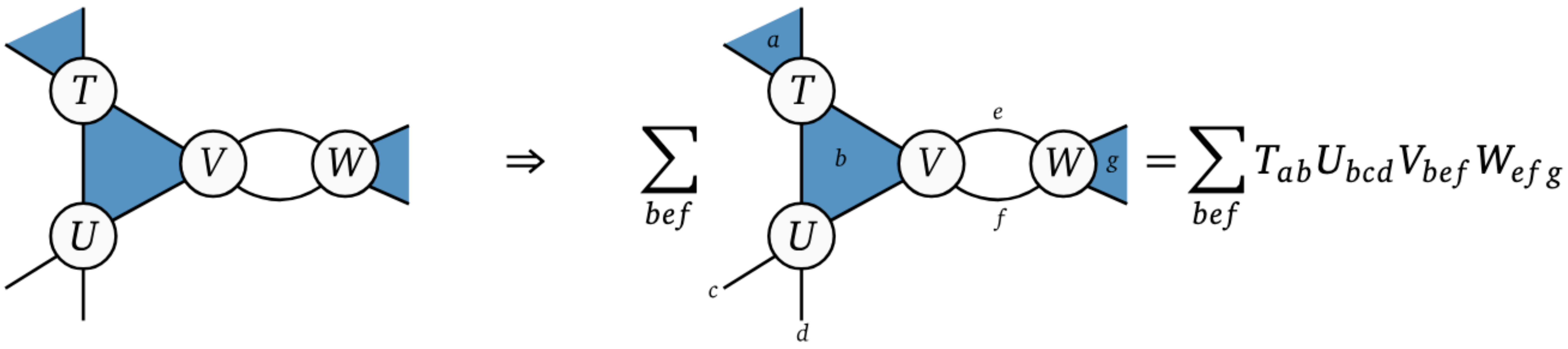
Open questions

- Systematic construction of gates and role of local transformations?
- Non-ergodicity/integrability?
- Role of space-time symmetry? Connection triunitarity DU2?
- Spectral form factors or other indicators of quantum chaos?
- How to generalize further?

Biunitary connections

Biunitary connections

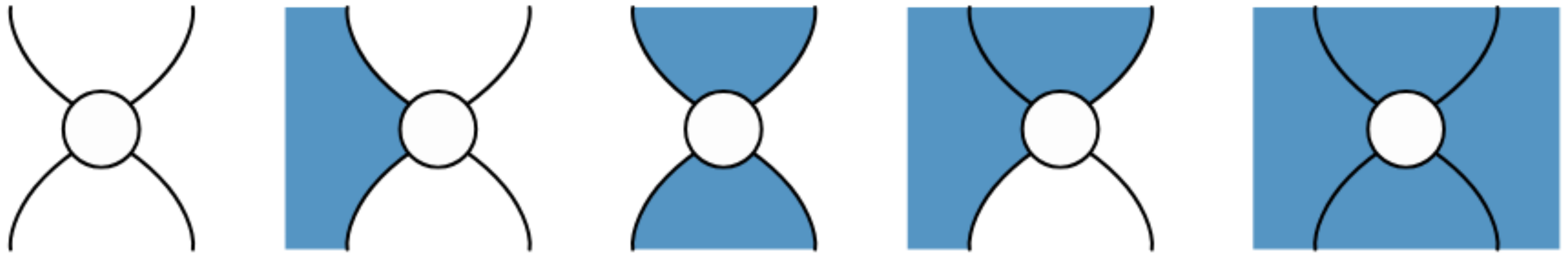
- Algebraic objects satisfying two notions of unitarity: vertical and horizontal
- Dual-unitary gates are specific instances of biunitary connections
- Represented in *shaded calculus*



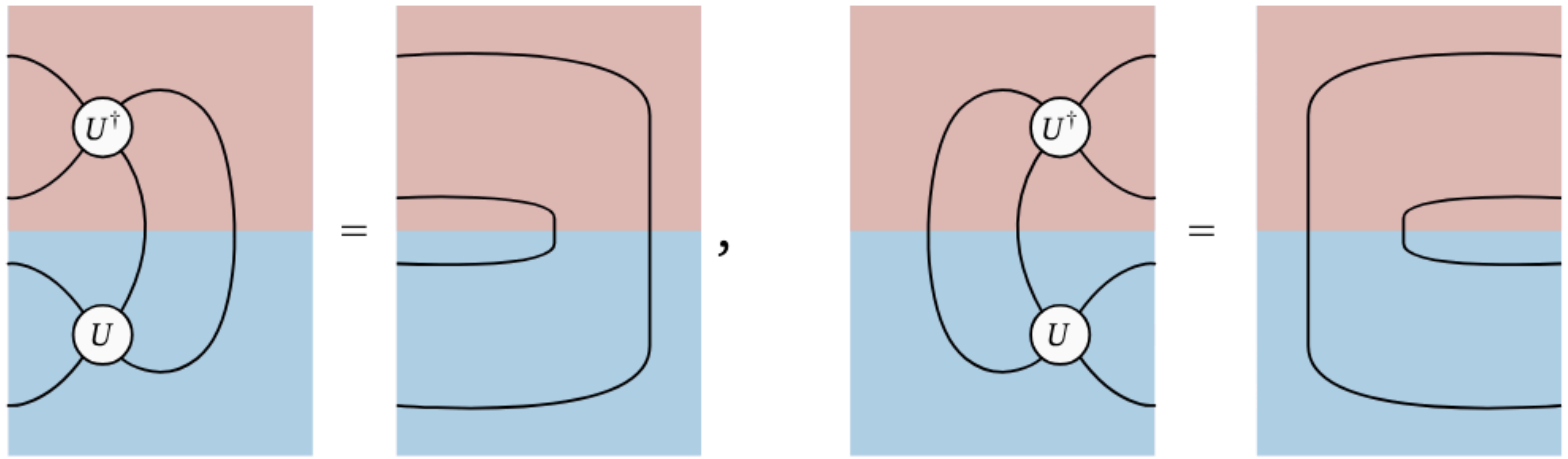
Ocneanu (1989), Reutter, Vicary (2016)

Biunitary connections

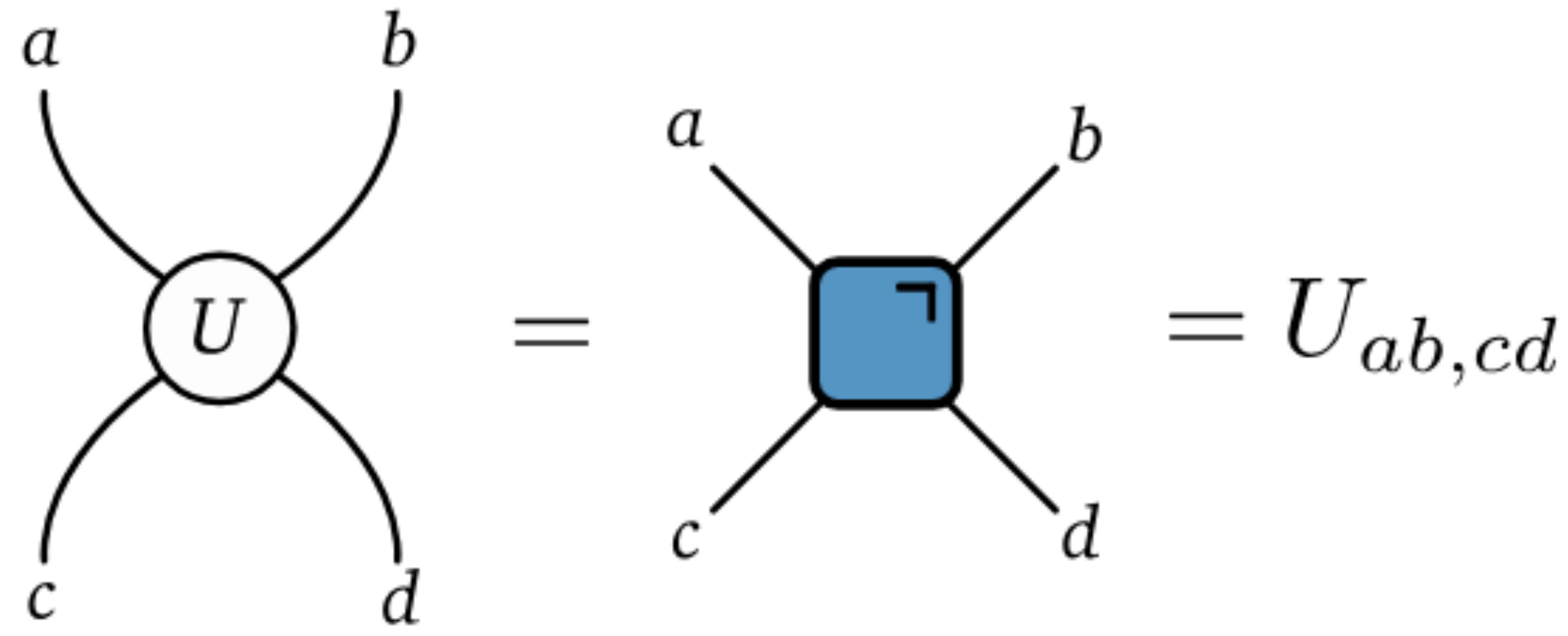
- In the plane have the following examples



- Unitarity conditions

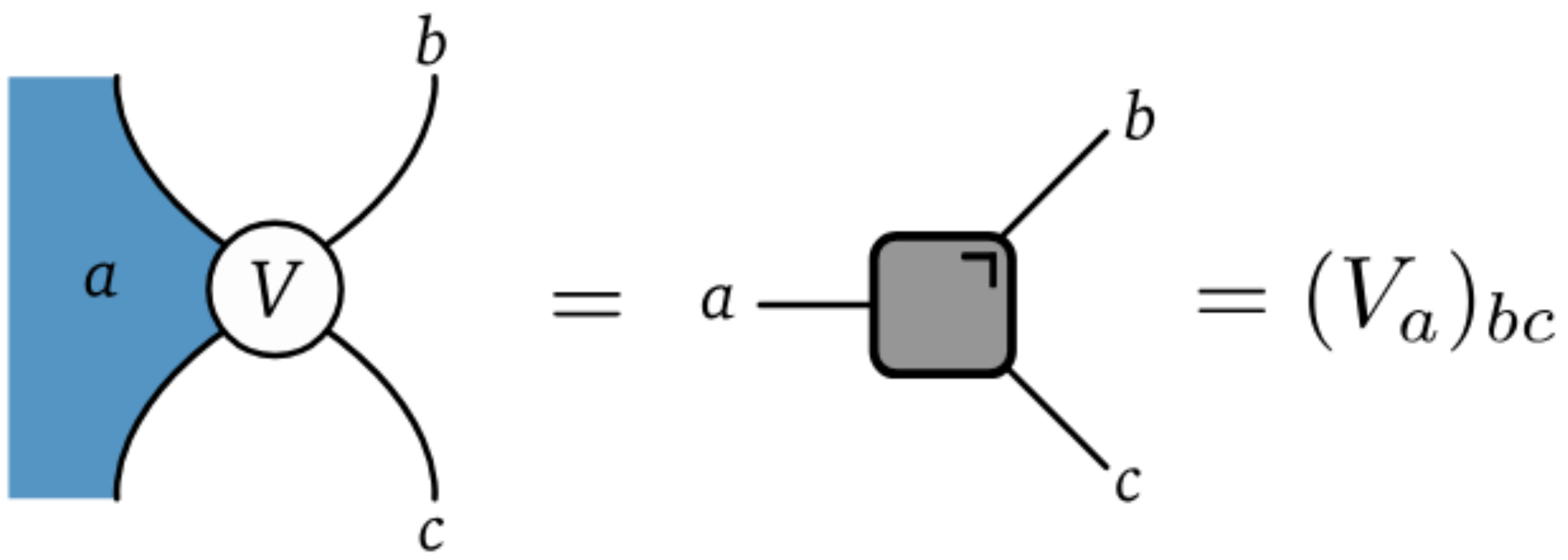


Dual-unitary gates



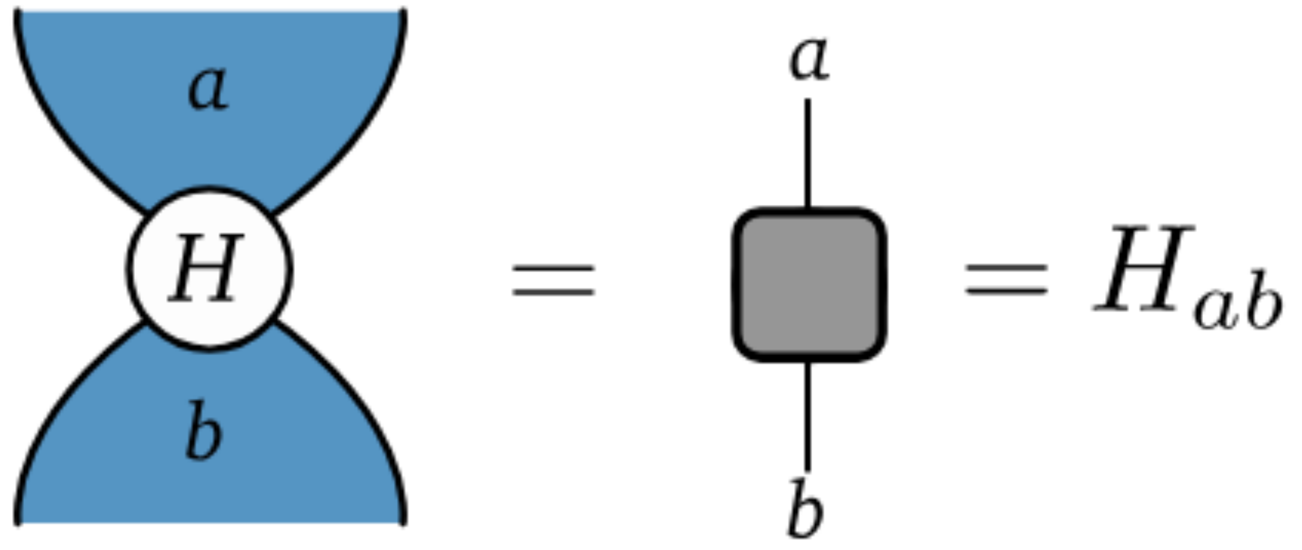
Unitary error bases

- Complete orthogonal family ($a = 1, \ldots, q^2$) of $q \times q$ unitary matrices



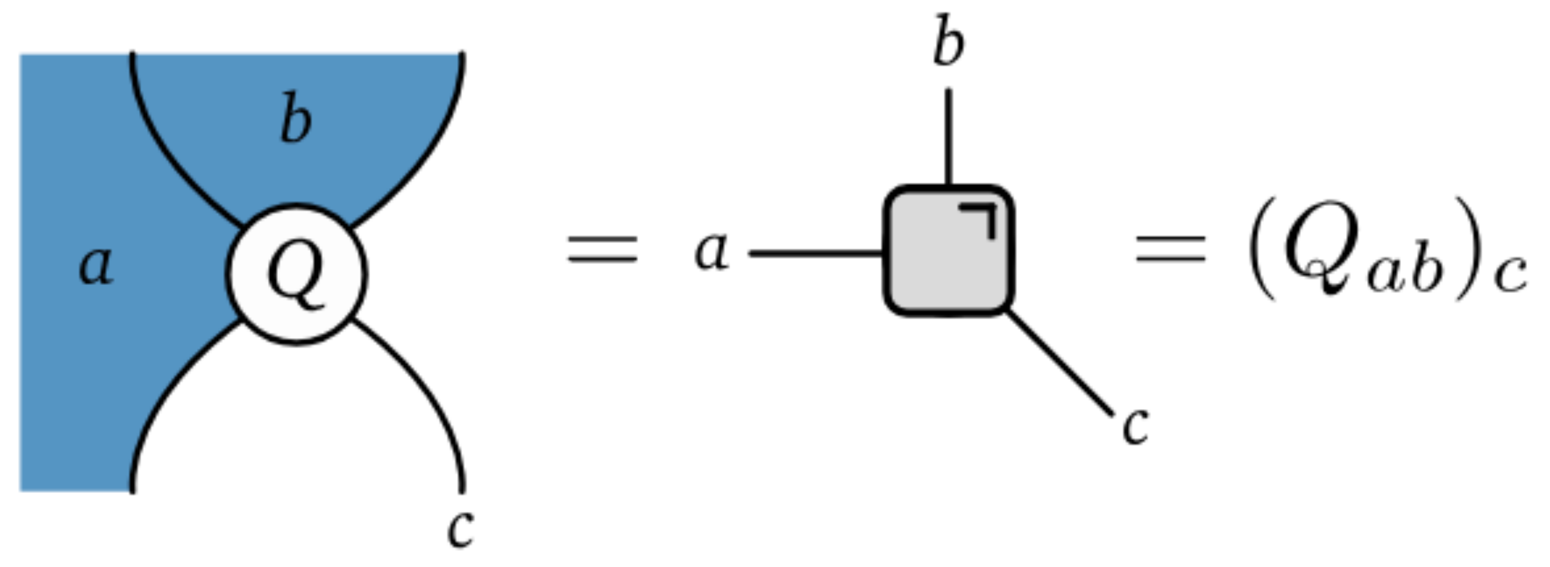
Complex Hadamard matrices

- Matrices proportional to unitary matrices with all entries having unit modulus



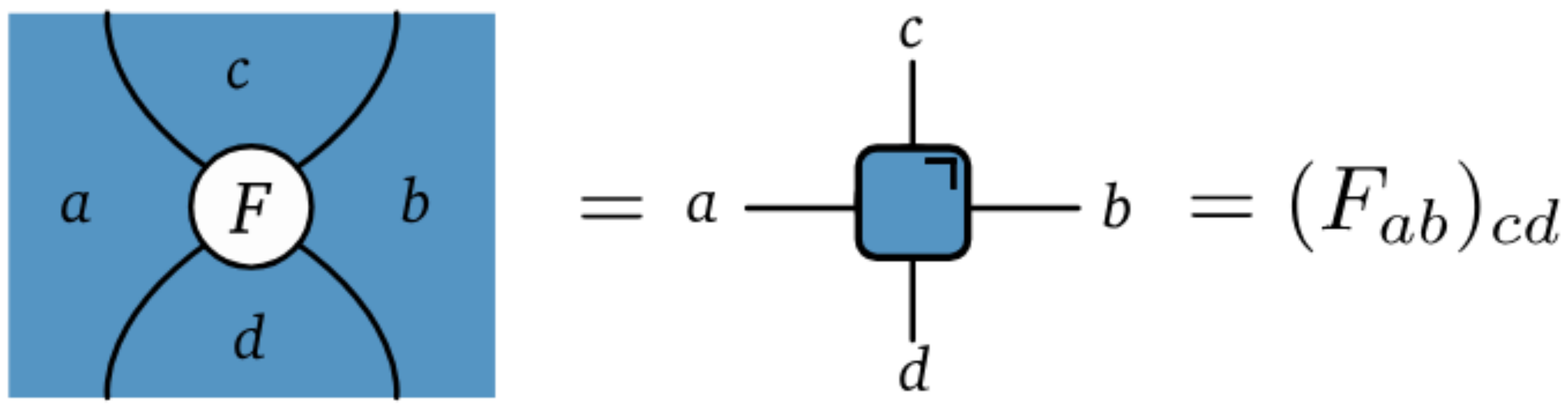
Quantum Latin squares

- $q \times q$ grid of states such that each row and each column forms a complete orthonormal basis



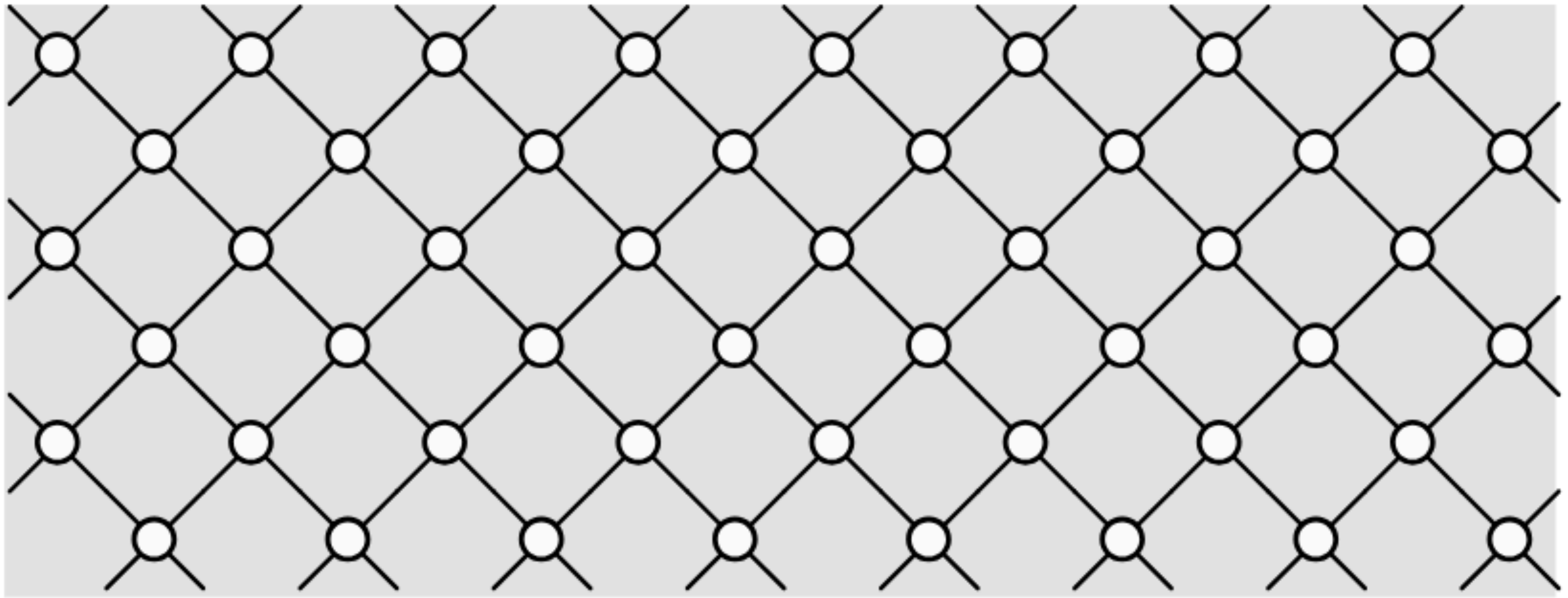
Dual-unitary interactions-round-a-face

- Double-controlled $q \times q$ unitary gates such that the gates remain unitary when control and physical legs are exchanged
- Also known as *quantum crosses*



Biunitary connections and solvable dynamics

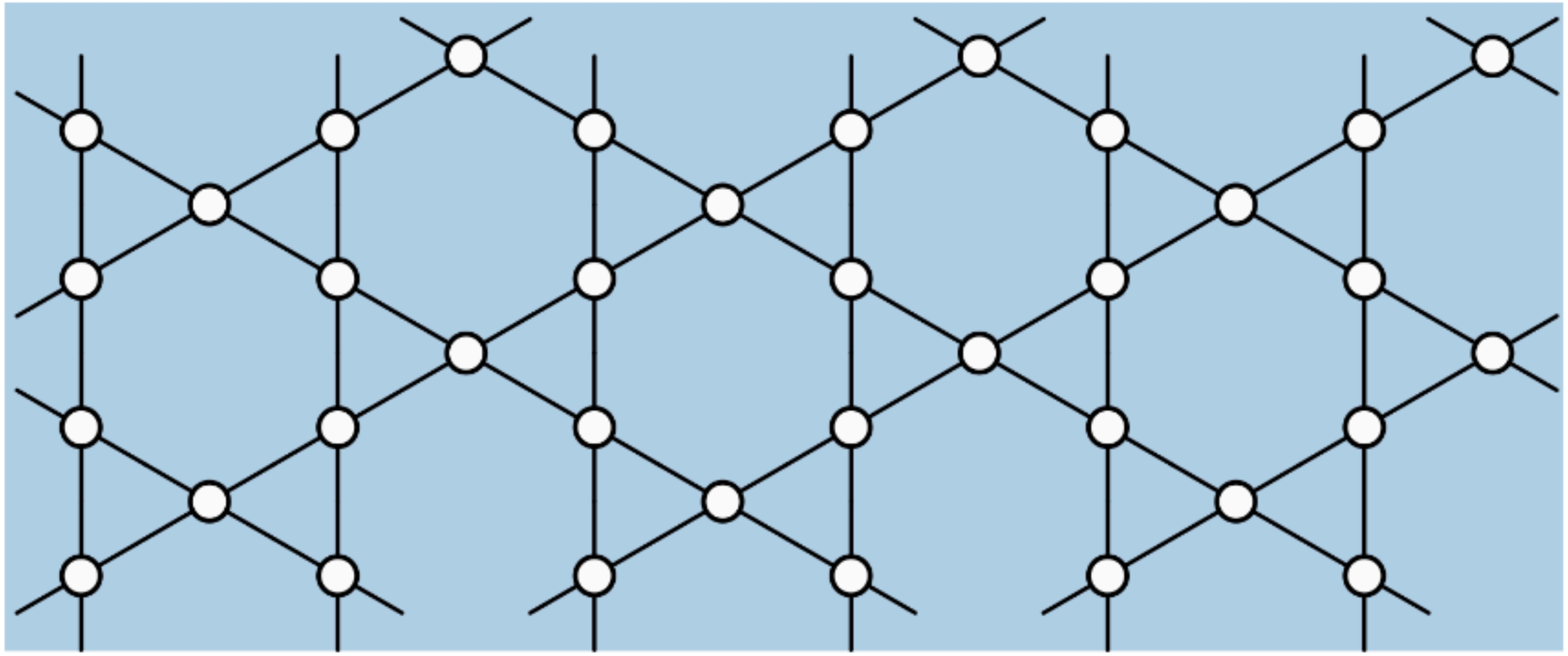
- A dual-unitary circuit can be represented as
- Central insight: the dynamics remains solvable for arbitrary shadings
- Unifies different approaches and enables systematic constructions



Claeys, Lamacraft, Vicary (2024)

Generalized dual-unitary dynamics and biunitary connections

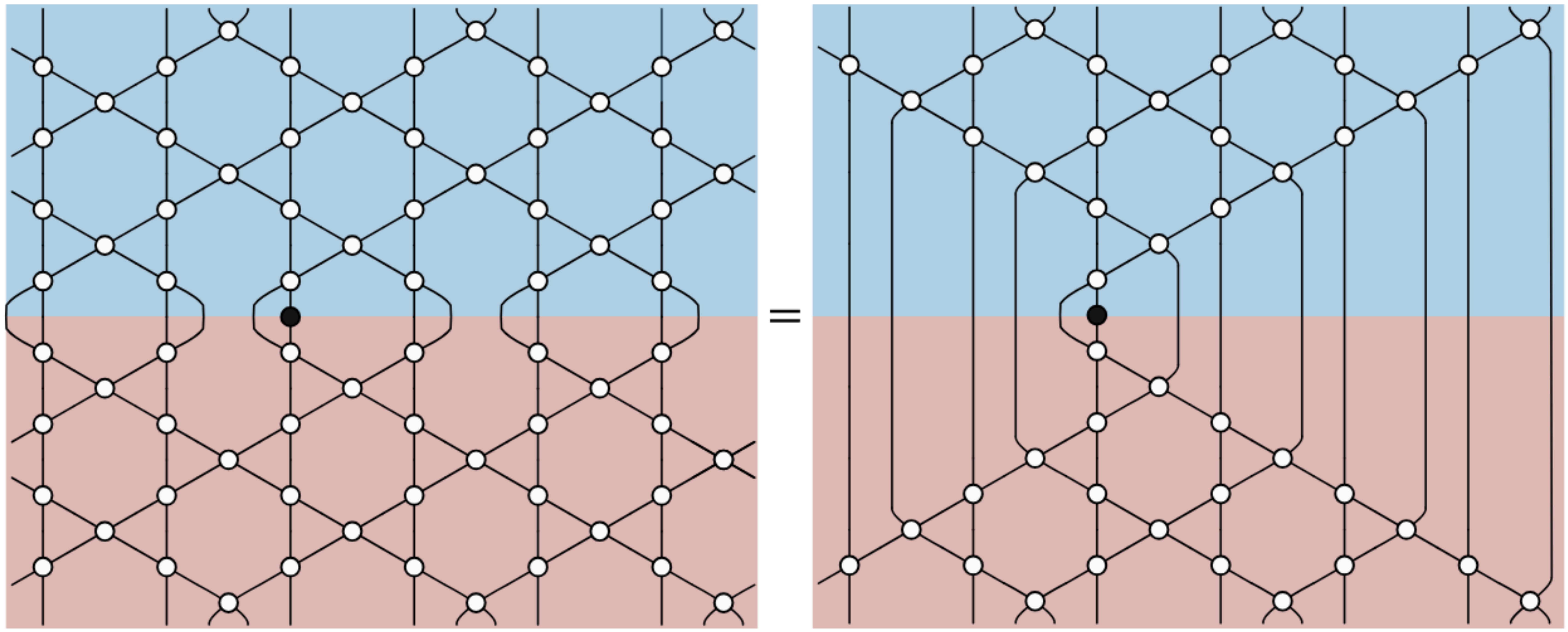
Biunitaries on the Kagome lattice

$\mathcal{U}(t) =$ 

Correlation functions

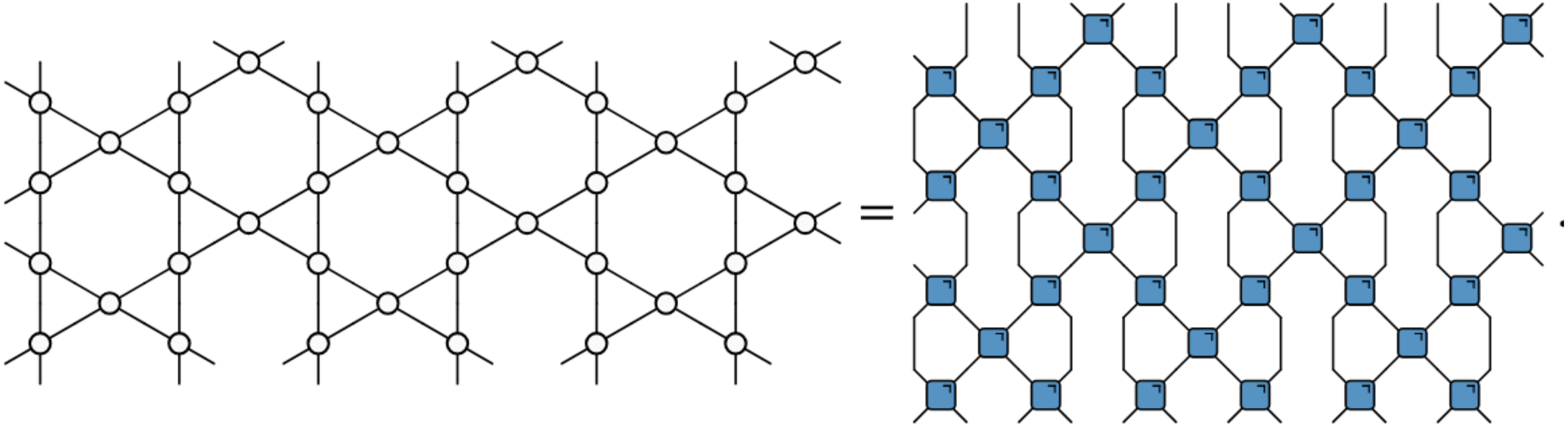
- Correlations non-vanishing only for $v = 0$ and $v = \pm 1$

$$\mathcal{U}(t)\sigma\mathcal{U}(t)^\dagger =$$



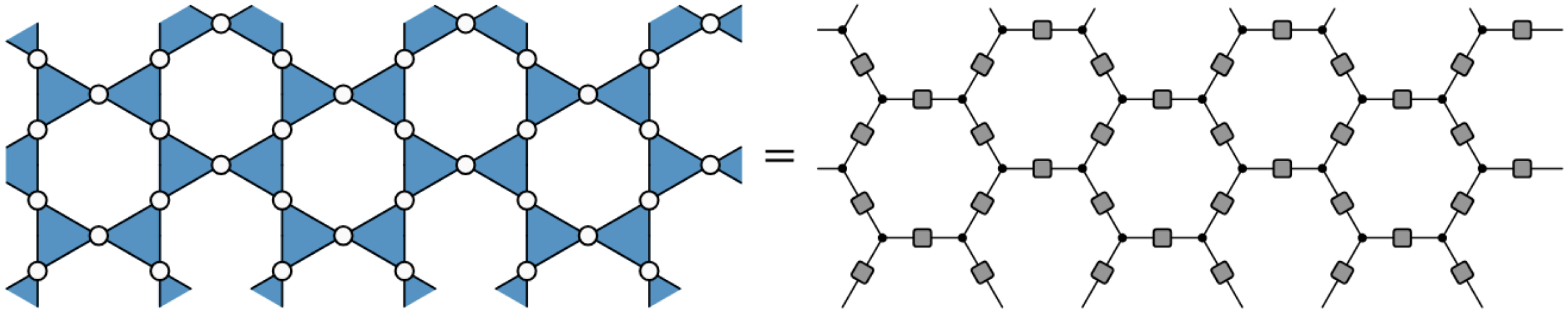
Examples

- Unshaded circuit: known to be triunitary



Examples

- Known representation in terms of Hadamard matrices
  → connects to CNOT, Floquet East model



Examples

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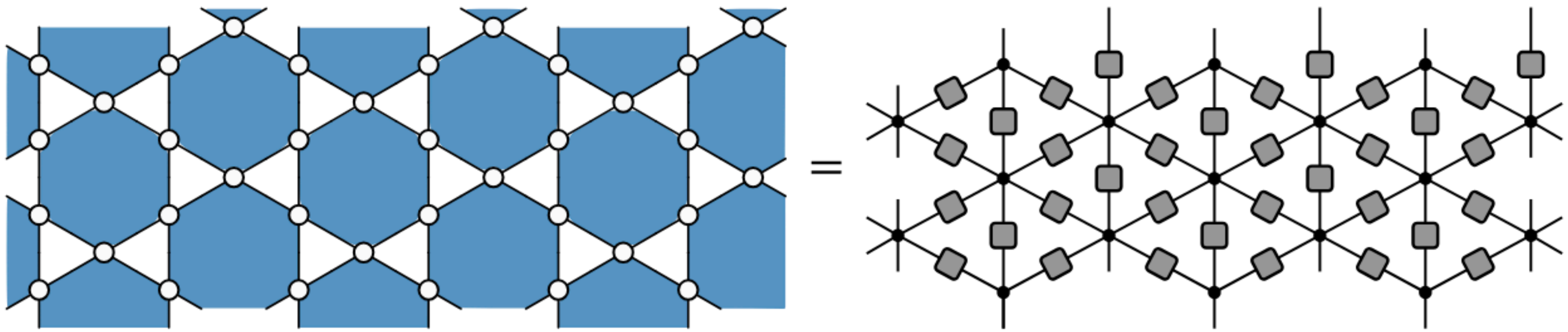
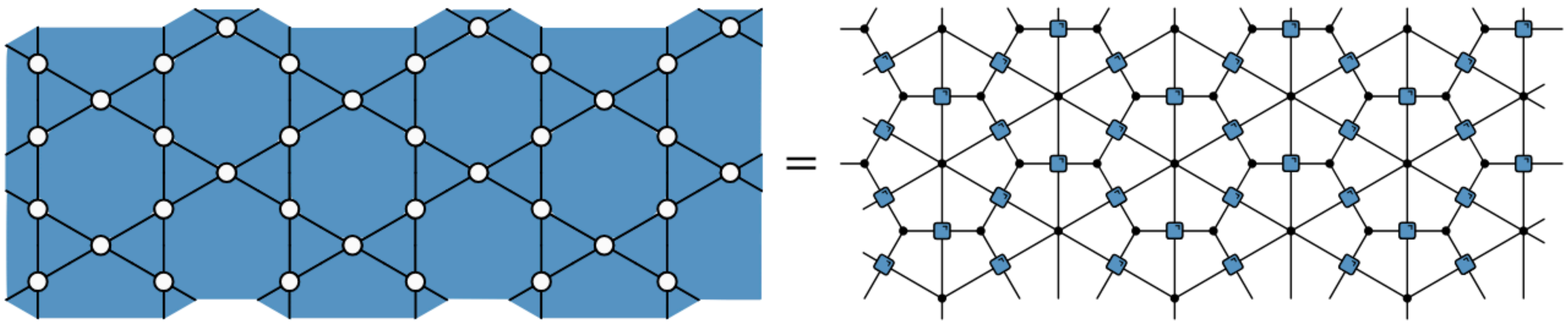


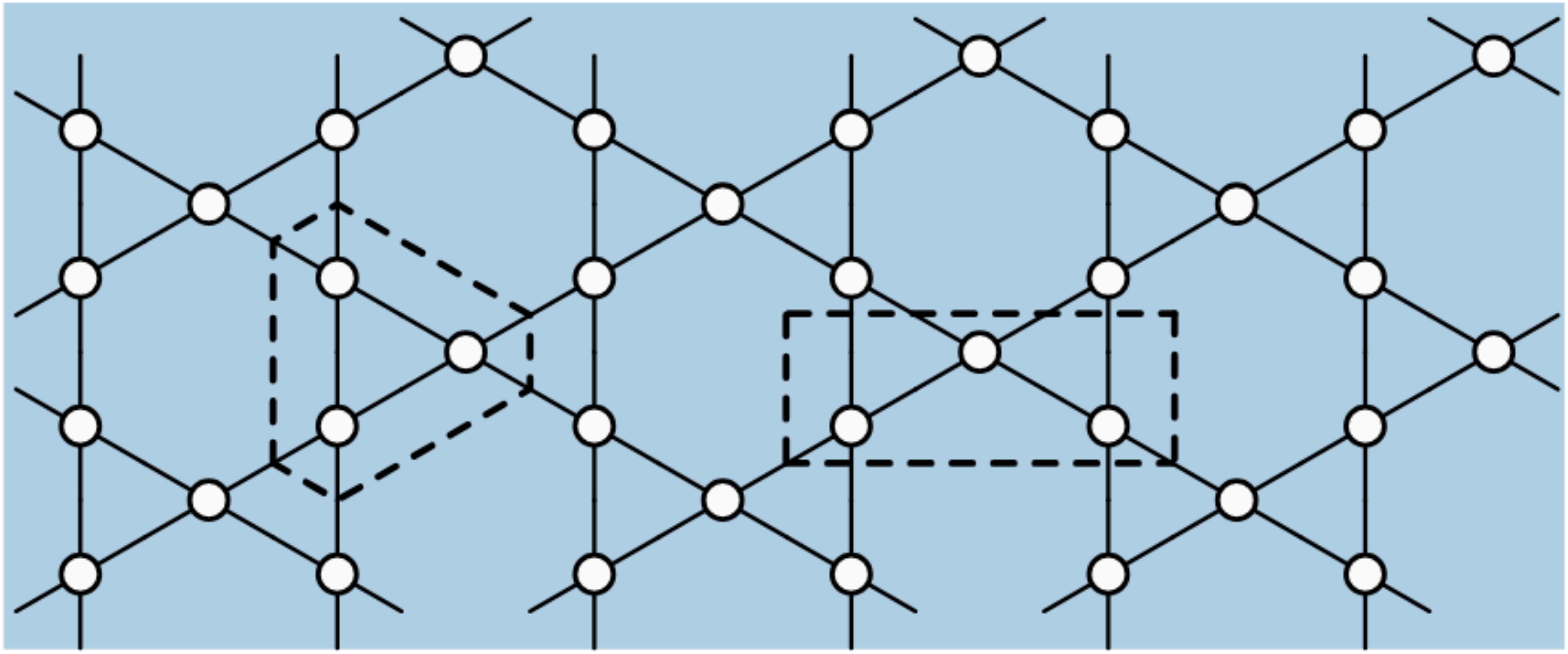
Examples

- Interactions-round-a-face



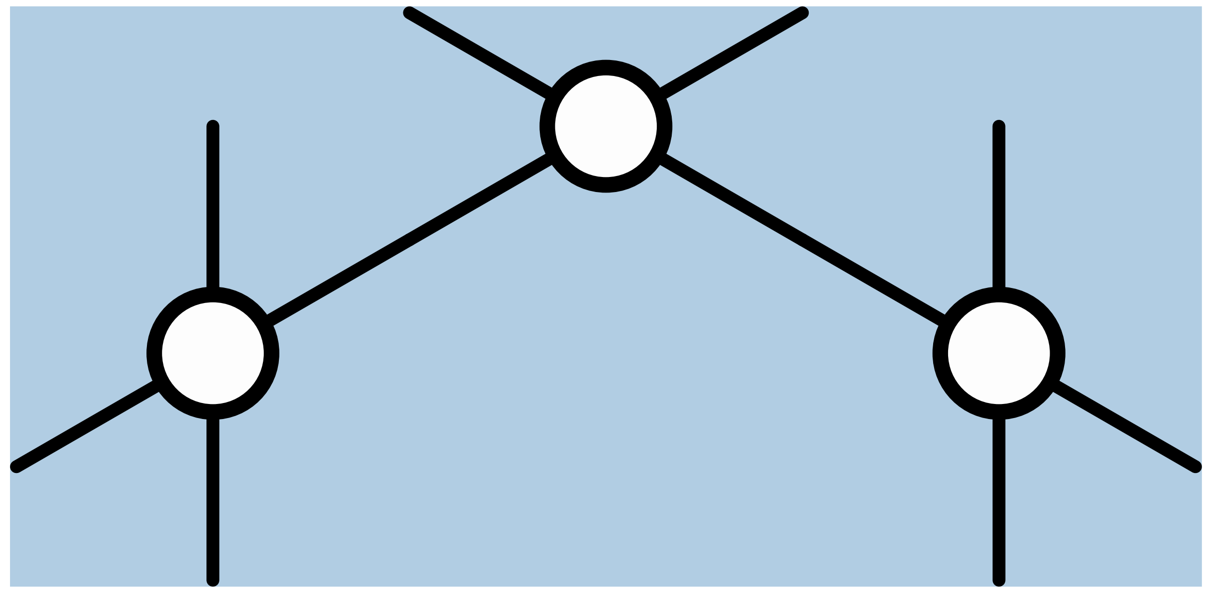
Identifying generalized dual unitarity

- Lattice can be generated by different building blocks

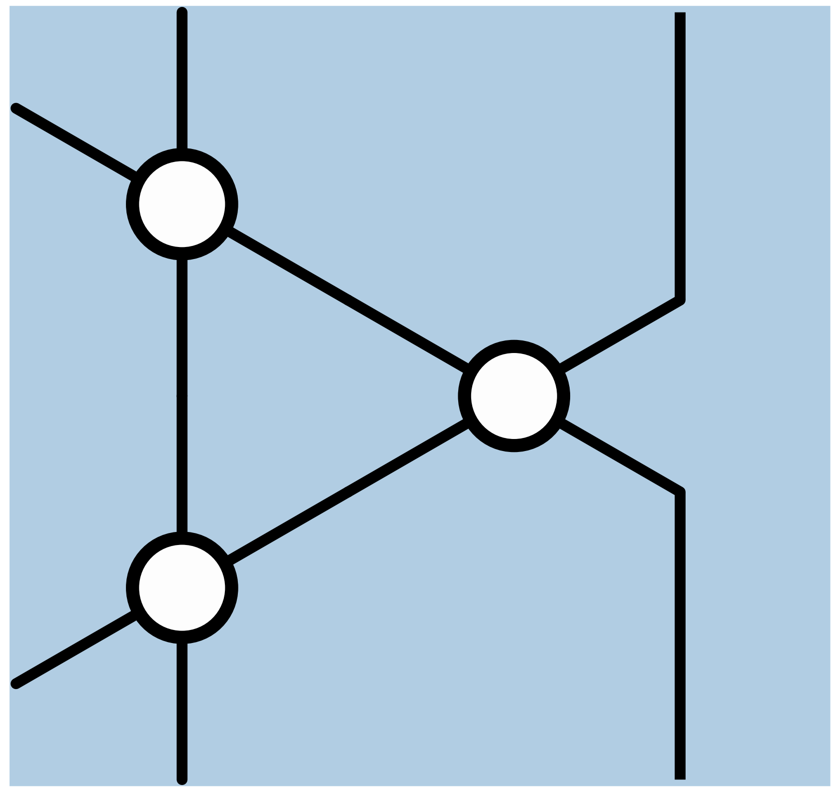


Identifying generalized dual unitarity

- Square satisfies generalized form of DU2 condition

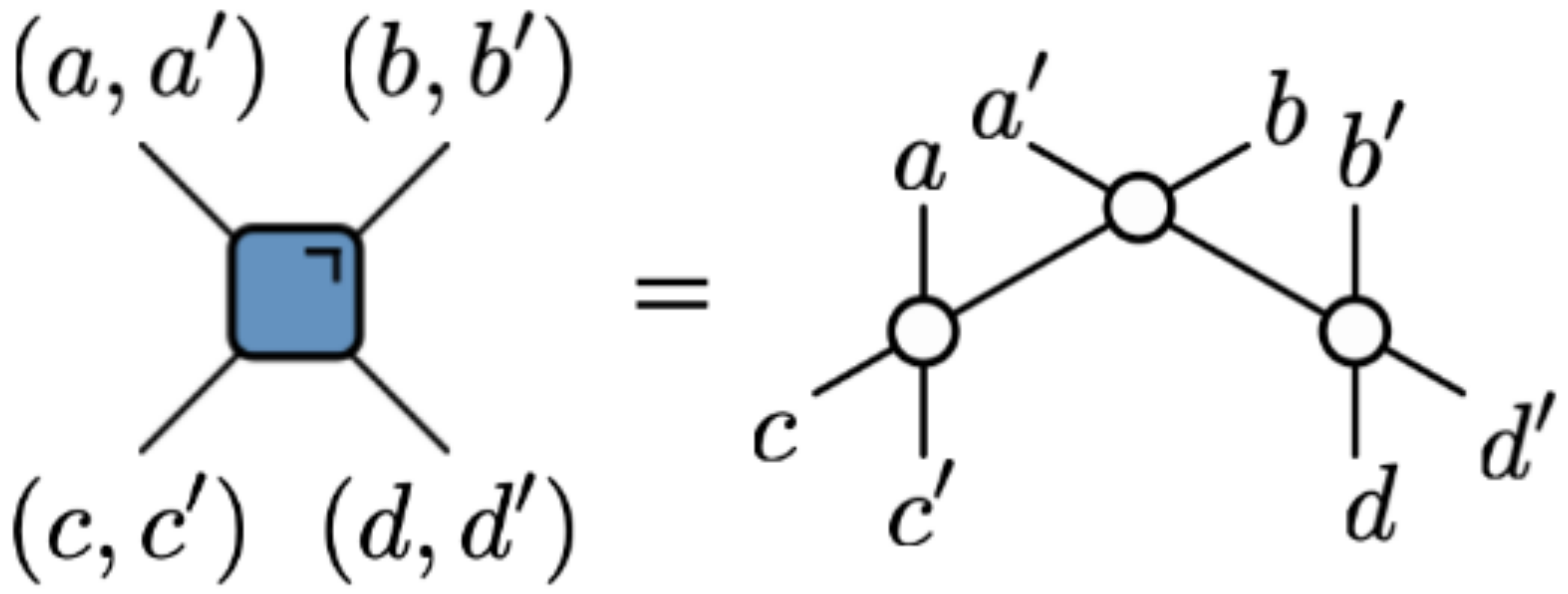


- Triangle corresponds to triunitary “gate"



Example

- Square satisfies generalized form of DU2 condition



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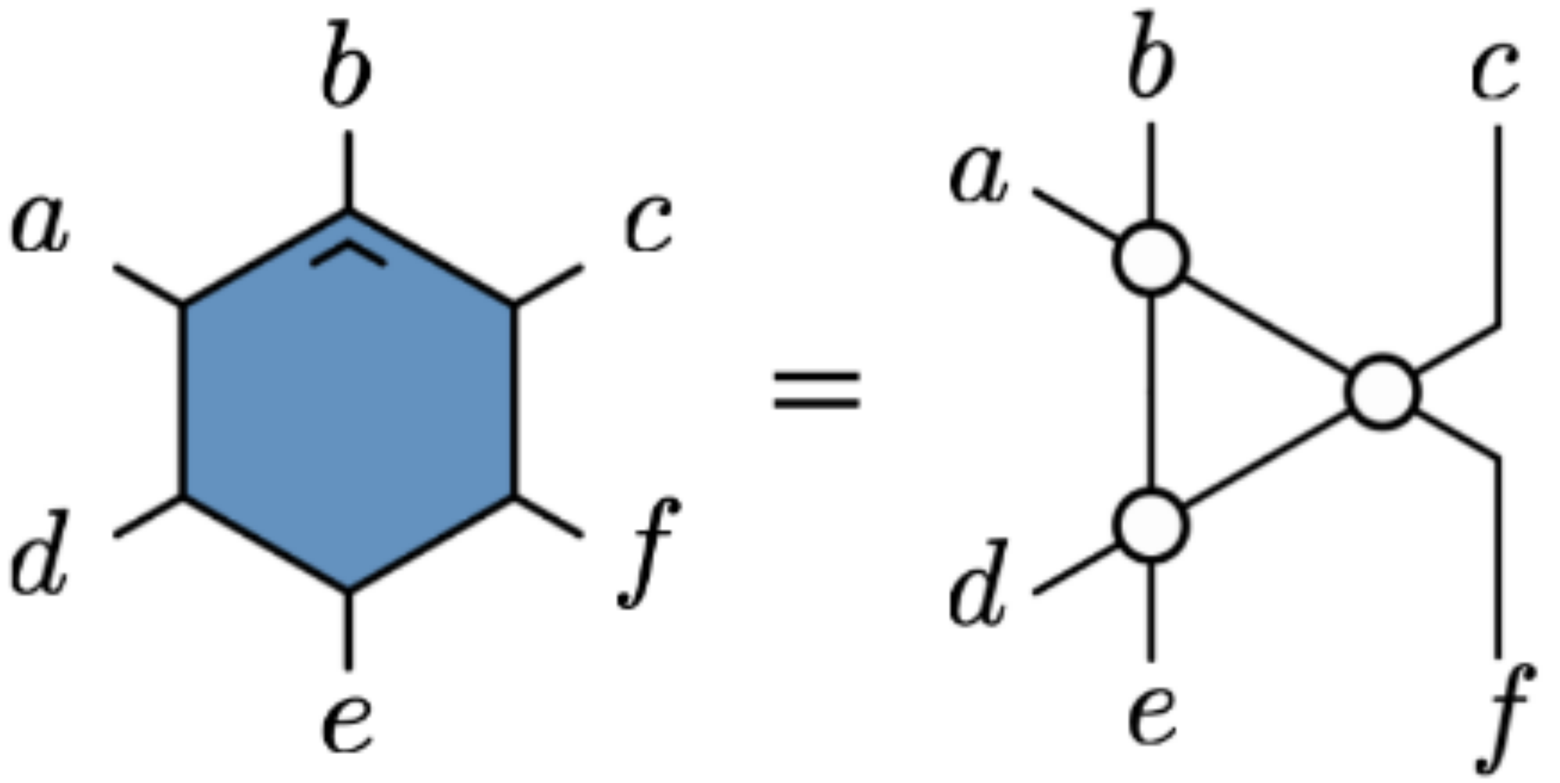
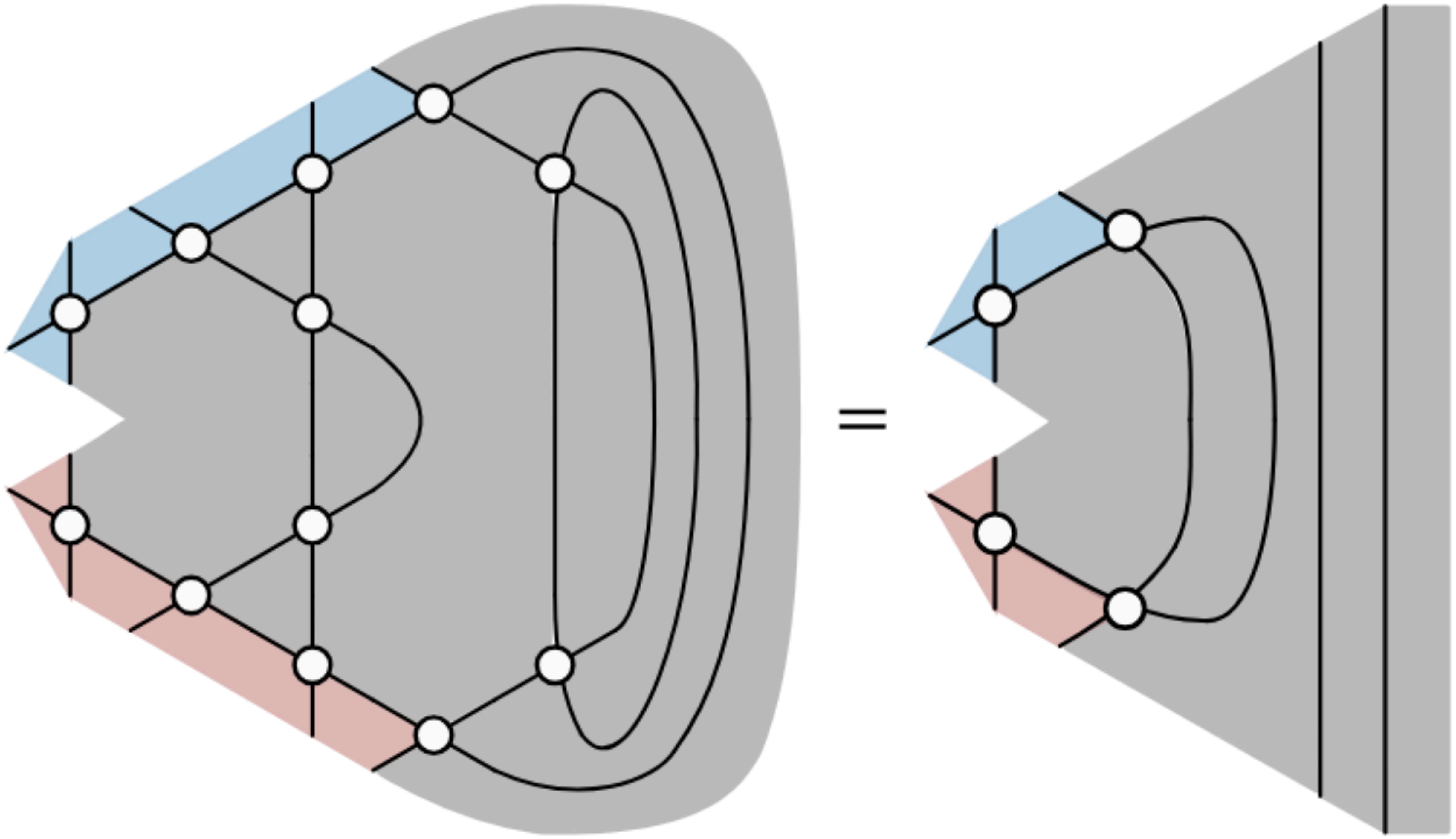


Identifying generalized dual unitarity

- DU2 and triunitarity conditions can be generalized to arbitrary shadings

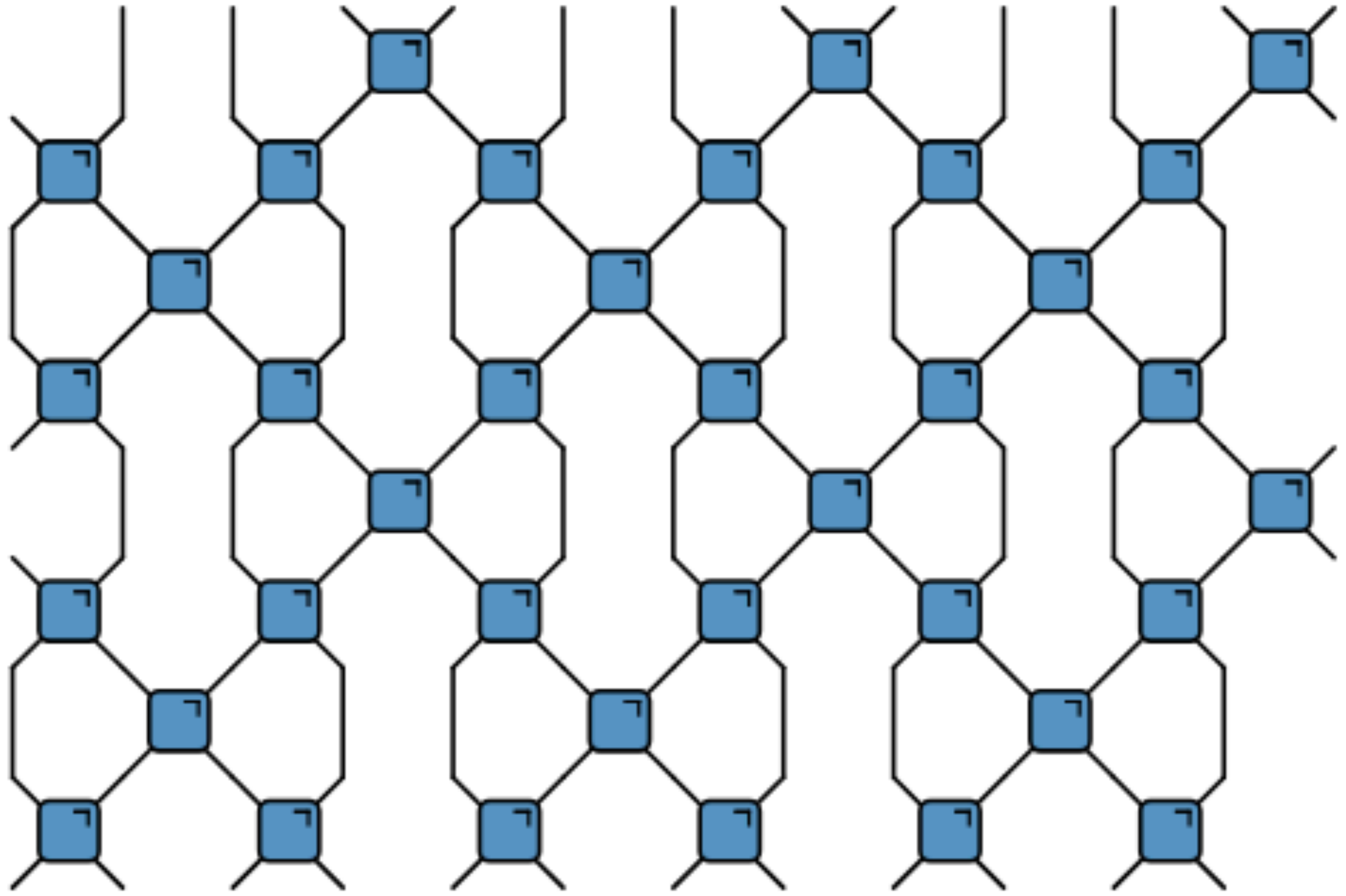


Non-ergodic DU2 gates

- Start with a DU gate that supports a soliton σ

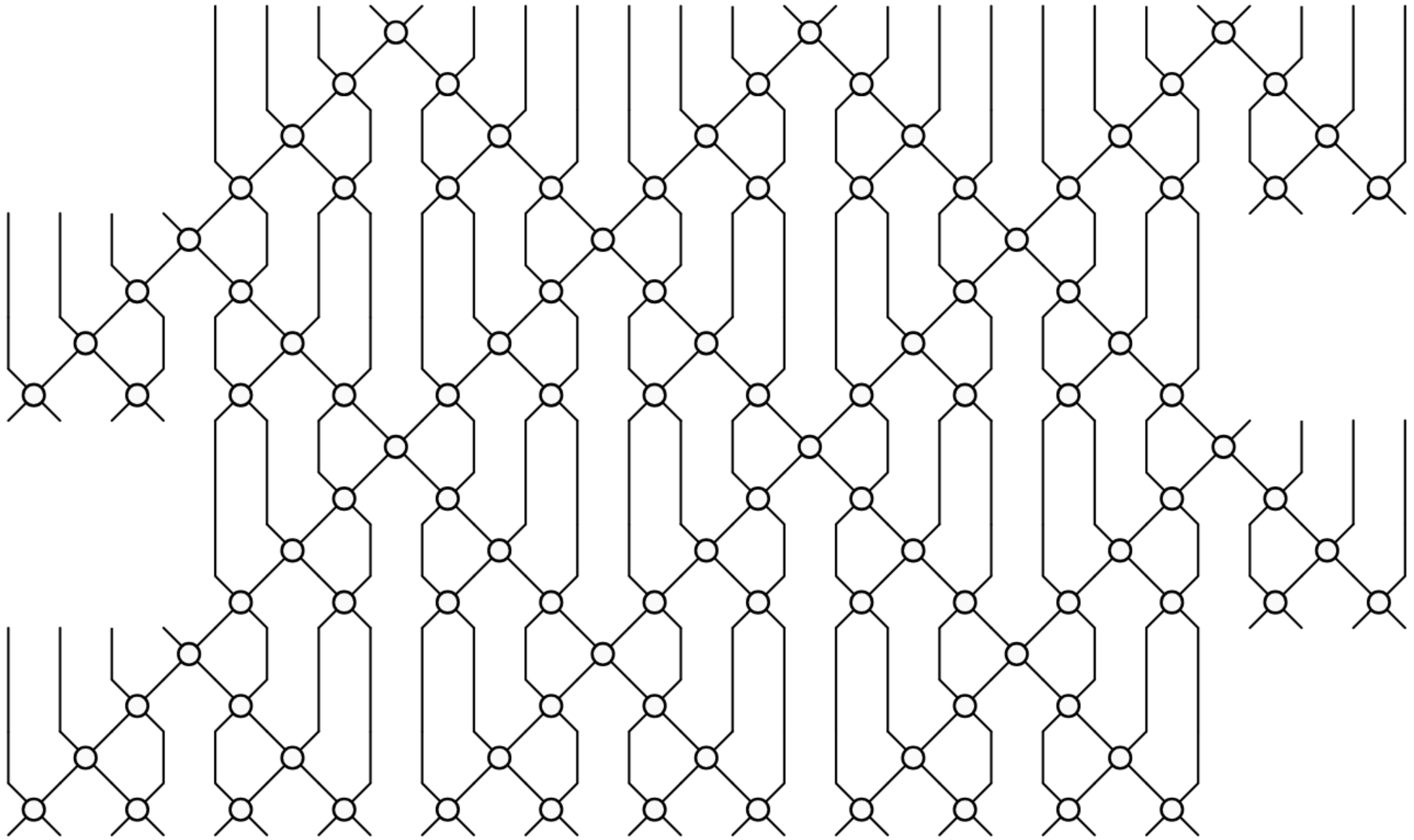
$$U(\sigma \otimes \mathbb{1})U^\dagger = \mathbb{1} \otimes \sigma, \quad U(\mathbb{1} \otimes \sigma)U^\dagger = \sigma \otimes \mathbb{1}$$

- Build a DU2 gate from three copies
  → supports two moving and two stationary solitons



Nested Kagome lattice

- Also defines a DU2 circuit with smaller entanglement velocity

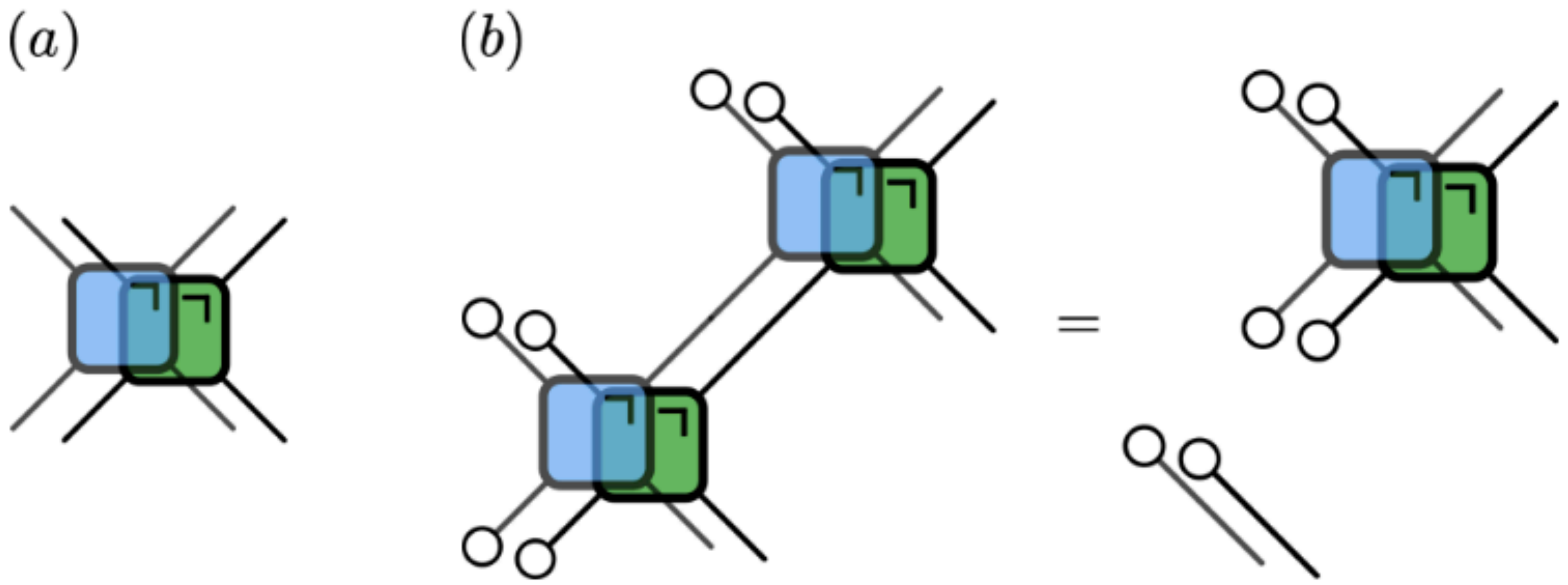


Multilayer constructions

Motivation

- The Kagome construction does not comprise DU2 gates with all possible entanglement velocities

- Compare to tensor product constructions → couple the layers?



Tensor product constructions

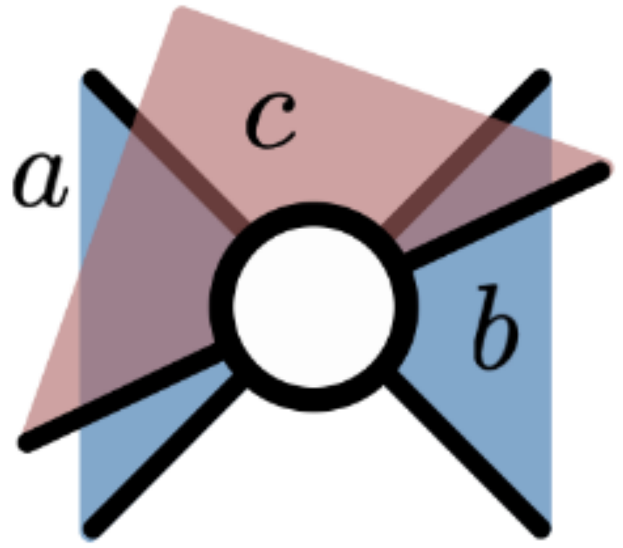
- Equivalent to asking: what are the possible Schmidt ranks for DU2 gates?
- Ingredients for tensor products:
  - Product gates: $\mathscr{R} = 1$
  - CNOT type: $\mathscr{R} = q$
  - DU: $\mathscr{R} = q^2$
- In composite dimension $q = q_1 q_2 \ldots q_n$: $\mathscr{R} = q_1^{\nu_1} q_2^{\nu_2} \ldots q_n^{\nu_n}$, $\nu_i = 0,1,2$

Rampp, Rather, Claeys (2024)

Multilayer biunitary connections

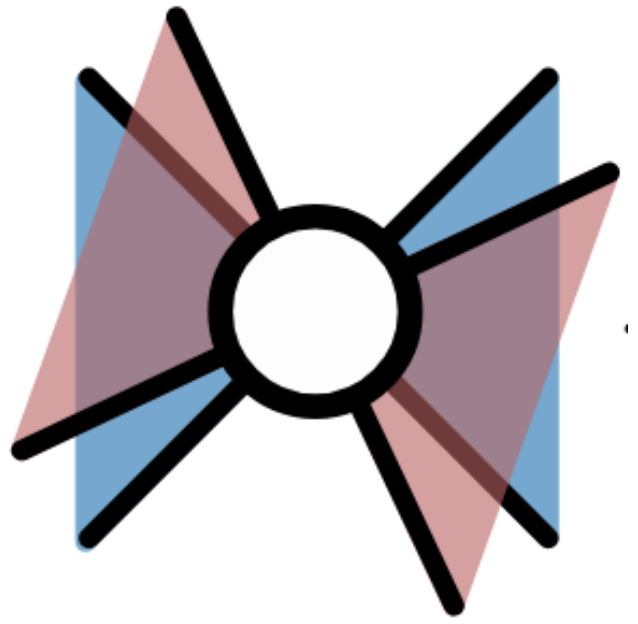
- Biunitary connections do not have to be drawn in a single plane
- E.g., a controlled family of complex Hadamard matrices

$H_{ab}^{c} =$



- Enables generalizing the tensor product

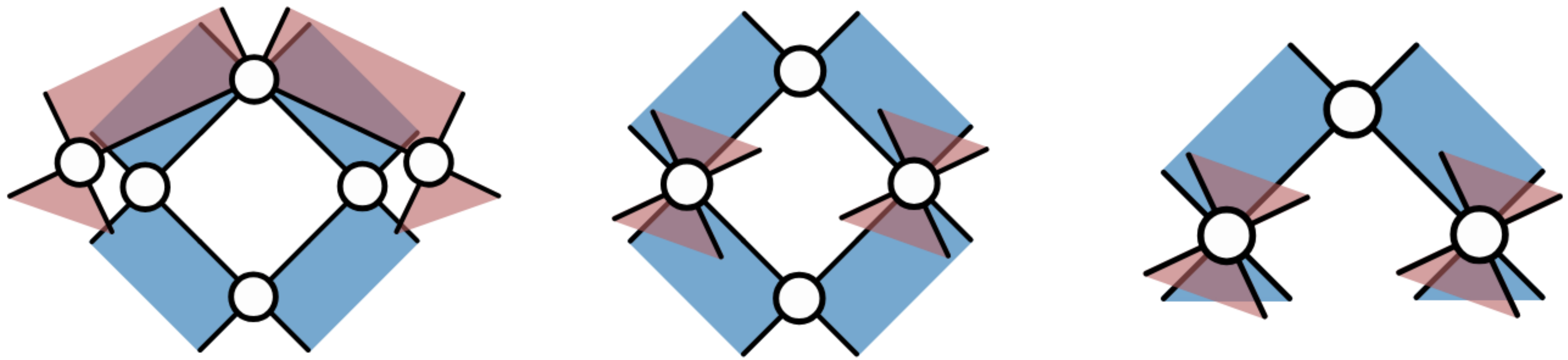
$H =$



Ocneanu (1989), Reutter, Vicary (2016)

Multilayer DU2 circuits

- Multilayer biunitary connections right language for coupling layers



Summary

- Biunitary connections on the Kagome lattice define a class of solvable dynamics comprising many examples of DU2 and triunitary circuits
- Unifies existing constructions
- Relates the patterns of correlations to symmetry directions in space time
- Clarifies the relation between DU2 and triunitarity