~~Measurement ...~~

~~Broad~~ ~~Broad~~ Broad goals arXiV: 2504.01264

- how much information is contained in measurements?

  * how does our knowledgge (the experimentalist) get updated post-measurements?

    - we should think about the knowledge of a certain property | meas.

      $P(\text{property} \mid \text{meas})$

      $P(O|M)$

    - we can also think about the full state

      $P(\text{state}|M)$

- this is an inference problem — follows Bayes' Rule
  - related to MIPT, charge sharpening, error correction
  - arXiV: 2504.02734 (RV, EM, SG), 2504.08888 (CVK, SWK, AL), 2504.12385 (ST, SG, HN, GYZ, MP)

- universal behavior of $P(S|M)$ in critical states, non-critical states, classical picture of charge sharpening. discussion of error correction.

Formalism

- Gaussian measurements; $\lambda = 1/2\Delta^2$ observable, O, true value O.

$$Z = \int_S e^{-H[S]}$$

$$P(M|S) = \exp\left(-\tfrac{1}{2\Delta^2}\sum_x (O_x - M_x)^2 + \underbrace{\ln d}_{\text{normalization}}\right)$$

$$P(M,S) = \exp(-H_{meas}[S,M])\,1/Z$$

$$H_{meas} = H + \frac{1}{2\Delta^2}\sum_x (O_x - M_x)^2 - \ln d$$

posterior

$$P(S|M) = \frac{P(S,M)}{P(M)} \qquad P(M) = \frac{\overbrace{\int_S e^{-H_{meas}[S,M]}}^{Z(M)}}{\int_S e^{-H[S]}}$$

$$= \frac{1}{Z(M)} e^{-H_{meas}[S,M]}$$

linear vs non-linear observables

$$\mathbb{E}_M\left[\langle S_x^2\rangle_M - \langle S_x\rangle_M^2\right]$$

$$\mathbb{E}_M\langle S_x^2\rangle_M = \langle S_x^2\rangle = \int_S S_x^2 e^{-H[S]}$$

$$\frac{Z}{Z(M)}\frac{e^{-H^1-H^2}}{Z^*} = \frac{P(S,M)P(S',M)}{P(M)} = P(M)\frac{P(S,M)P(S',M)}{P(M)^2}$$

$$\begin{aligned}\mathbb{E}_M\langle S_x^2\rangle_M &= \int dM\, P(M)\langle S_x^2\rangle_M \\ &= \int dM\,ds\, P(M)\,P(S|M)\,S_x^2 \\ &= \int ds\left[\int_M P(M)P(S|M)\right]S_x^2 \\ &= \int ds\, P(s)\, S_x^2\end{aligned}$$

$$\begin{aligned}\mathbb{E}_M\langle S_x\rangle_M^2 &= \int_M P(M)\,\langle S_x\rangle_M^2 \\ &= \int_M P(M)\int_{S,S'} P(S|M)\,P(S'|M)\,S_x S_x' \\ &\neq \int_S ds\, P(S)\,(\text{observable})(S)\end{aligned}$$

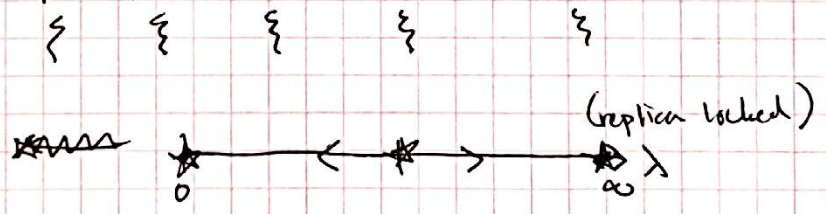
Replica $\frac{1}{Z}\int_{M,\dots} e^{-\sum_{a=1}^{N} H_{meas}[S^a, M]}$

Examples

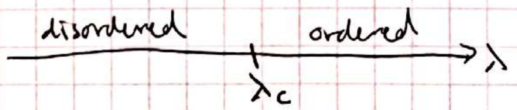
a) Ising 2D at criticality
measure $\varepsilon_x = S_x S_{x+1}$

i) weak measurements marginally irrelevant

ii) strong measurements ; know location of DW; state specified up to flip.



b) Ising $T=\infty$ 2D



$$H = \lambda\sum_{\langle xy\rangle}\sum_{a\neq b}(S_x^a S_y^a)(S_x^b S_y^b) \qquad (\mathbb{Z}_2\ \text{SW}\to\text{SSB})$$

$$\mathbb{E}_M\langle S_x S_y\rangle^2 \to \text{const.}$$