Many-Body Cages

Tan & Huang	2504.07780	10.04	1
Ben-Ami, Markus Heyl & Roderich	2504.13086	17.04	2
Serbyn	2504.17627	24.04	3
Chergne & Frank P.	2504.20987	29.04	4

4 **Model:** (pbc)

$$H = \sum_{i \text{ even}} X_i \left(P_{i-1} + P_{i+1} \right) + \sum_i X_i \left(P_{i-1} P_{i-2} P_{i-3} + P_{i+1} P_{i+2} P_{i+3} \right)$$

$$P_i = \frac{1 - Z_i}{2}$$

Dynamics:

1. term o o o o o o o o o o

→ flip o if it has one full neighbour

2. term o o o o o o o o o o o o

→ flip green if it has 3 consecutive full neighbours

→ Dynamical constrained model
(see also PXP model, quantum hard disks, U(1)-link model, quantum East model ...)

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Fock – Space:

- Can map many-body Hamiltonian to single-particle hopping on Fock space with weird lattice (isomorphism)

- isomorphism $\Rightarrow$ both Hamiltonians have the same spectrum

- Tan & Huang: expect QMBS to have low entanglement $\to$ choose low entangled basis

- bipartite lattice if imbalance $\Rightarrow$ $\exists$ zero modes $\to$ generated by symmetry

$$H = \begin{pmatrix} 0 & \overbrace{M}^{\text{dim } a} \\ M^{+} & 0 \end{pmatrix} \}\ \text{dim } b$$

if dim $a \neq$ dim b $\Rightarrow$ $\exists$ zero modes
$\to$ find by $M|\psi\rangle = 0$

(e.g. Chiral symmetry generates bipartite lattice $P = \prod_i Z_i$

$\Rightarrow HP + PH = 0$

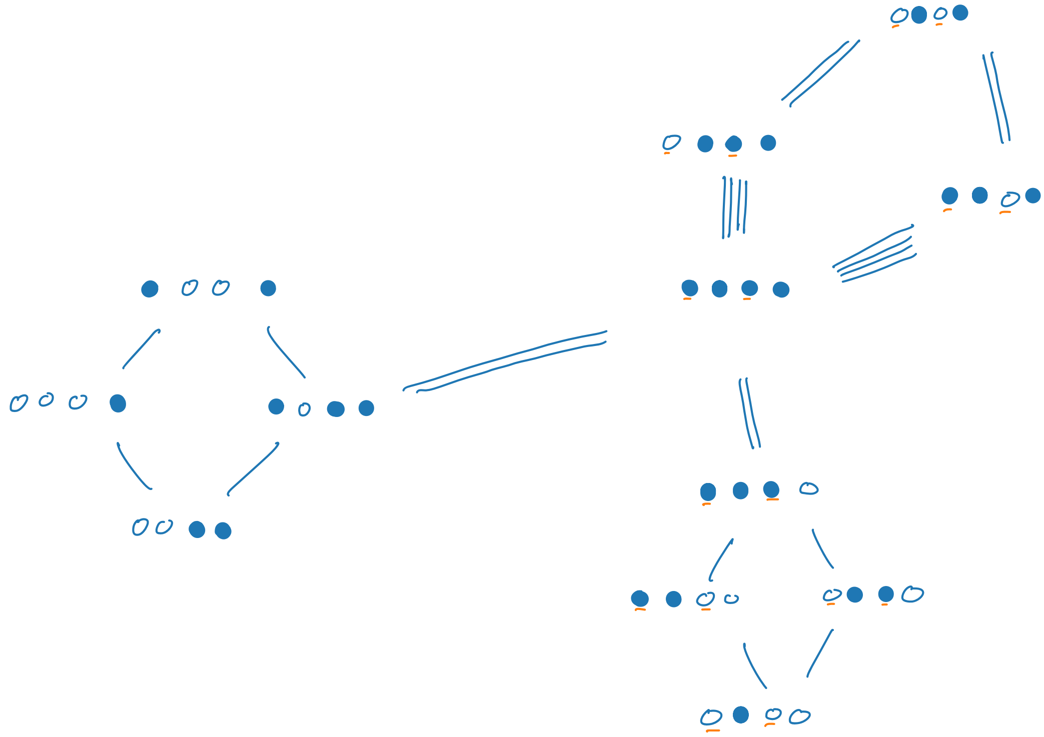
$\Rightarrow H|\psi\rangle = 0 \Rightarrow H P|\psi\rangle = 0$

$\Rightarrow |\psi\rangle, P|\psi\rangle$ both 0 states

$\Rightarrow P$ diagonal in 0 space

$\Rightarrow$ 0 states live either in $P=+1$ or $P=-1$ $\Rightarrow$ bipartite

Next: Construct Fock-space Graph $L = 4$:



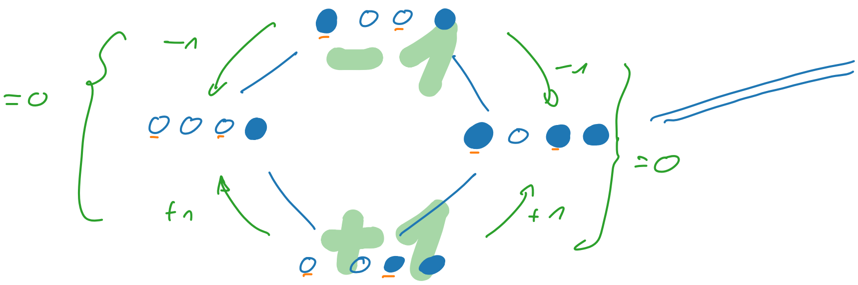
Missing

○○○○
○○●○
●○○○
●○●○

→ all not connected to anything else

Fock-Space Cages   Cheng & Pollmann

Example:



Consider

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\circ\circ\bullet\bullet\rangle - |\bullet\circ\circ\bullet\rangle\right)$$

$$\Rightarrow \quad H|\psi_0\rangle = 0$$

- decoupling due to interferences, state itself is part of large Krylov-state network
- Huang: localization due to local lattice topology

- zero energy modes localized on sublattice
- form closed loop (cage)

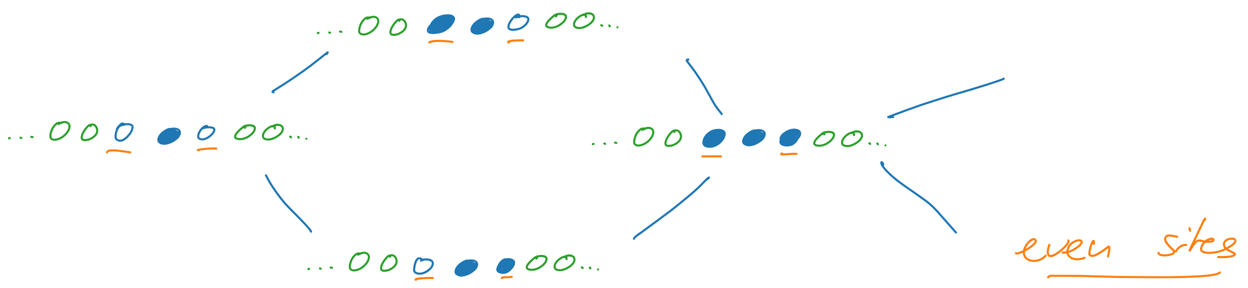
Huang

- similarity to single-particle localization (Kagome)
- low entanglement
- dubbed interference caged quantum many-body scars

Thermodynamic limit:

Now: Same holds for arbitrary L:

trivial example:

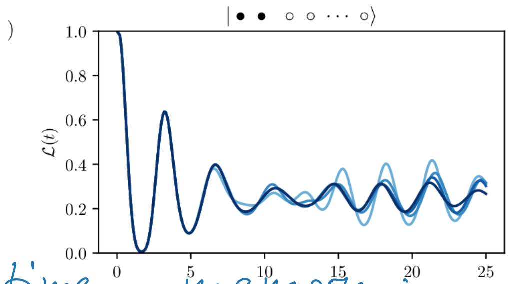


↳ can also be combined to give larger 0 mode spaces

⇒ Many Body Caged zero energy states

⇒ One example of Tree Grafting

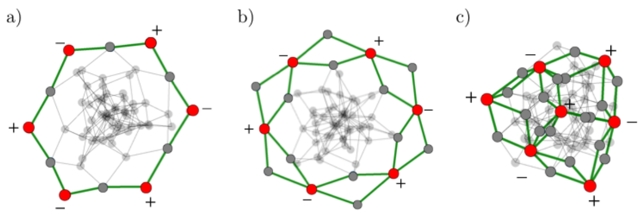
Dynamics:



- long time memory:

start with $|\varphi\rangle = |\dots 0 0 0 \, \bullet \, \bullet \, 0 0 \dots\rangle$

→ $|\langle\varphi|\varphi(t)\rangle|^2 > 0$ also for large times



tJ

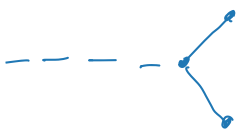
Generalization:

1. multi-site caged states: $H = \sum_i x_i (p_{i-1} + p_{i+1})$

$\hookrightarrow \sum_i (-1)^i |\dots 0_{i-1} \bullet_i 0_{i+1} \dots\rangle$

2. Tree grafting (Roderick ...)

- tree generating close to percolation transition
- localized states on dangling subtrees yield large eigenvalue degeneracy

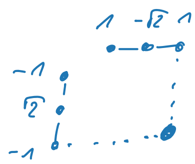


$\rightsquigarrow$ set of nodes, eigenstates have 0 weight on connected node → eigenstates still exist when connected to other stuff

e.g.

$\hookrightarrow \psi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}; \quad \psi_{\sqrt{2}} = \frac{1}{2}\begin{pmatrix}1\\\sqrt{2}\\1\end{pmatrix}; \quad \psi_{-\sqrt{2}} = \frac{1}{2}\begin{pmatrix}1\\-\sqrt{2}\\1\end{pmatrix}$

Can graft



→ Eigenvalue $-\sqrt{2}$, localized

- Huang Mathematical expression:

$$\begin{pmatrix} A_{G'} & K \\ K^T & A_{G-G'} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ 0 \end{pmatrix} = \begin{pmatrix} A_{G'}\mathbf{x} \\ K^T\mathbf{x} \end{pmatrix} = \begin{pmatrix} \mu\mathbf{x} \\ 0 \end{pmatrix}, \qquad (4)$$

$K^T x = 0$: interference-cage condition
(also used by Chayes)

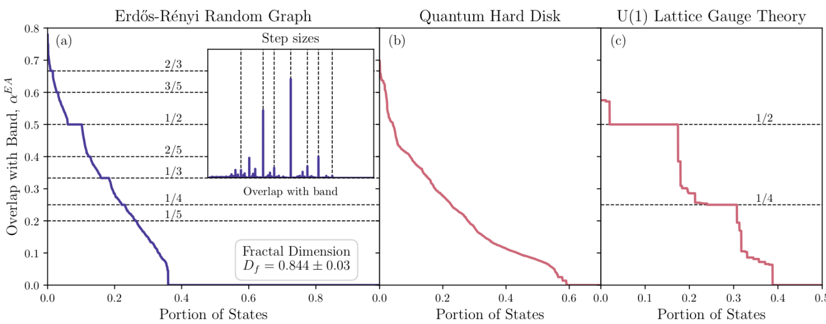
↳ **Lemma III.1.** *Given a subgraph G', if for every vertex v_i in the inner boundary of G', there exists at least one automorphism partner $\sigma(v_i) \in G'$, such that v_i and $\sigma(v_i)$ are uniformly connected to the same vertices $\{u_i\}$ on the outer boundary of G' (which stays unchanged under σ), then G' possesses localizable eigenvector(s).*

Lemma III.2. *Given a subgraph G', if every orbit in the inner boundary of G' is connected to a common orbit $\{\sigma(u)\}$, which forms the outer boundary of G', then G' possesses localizable eigenvector(s).*

- beyond bipartite-ness
- severall possible specific eigenvalues
- tree occurance $\propto \exp(-$ tree size$)$
- characterize via $\alpha^{EA}(e, \mathcal{E}) = \Big| \langle \phi_e | \underbrace{\sum_{n | E_n = \mathcal{E}} | E_n \rangle}_{\text{still eigenstate with eigenvalue } \mathcal{E}} \Big|^2$

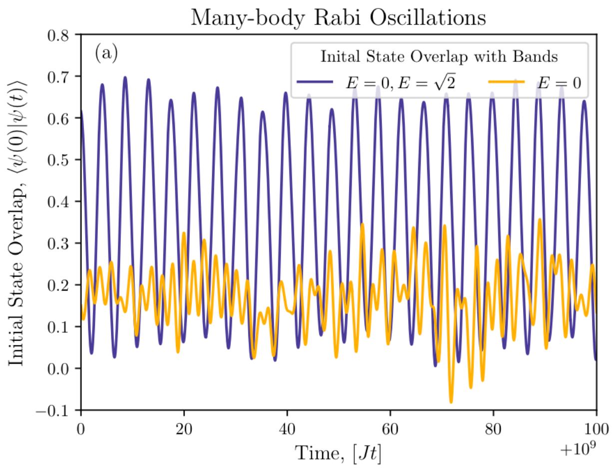
  e: fock space site; $\mathcal{E}$: eigenenergy

(does this make sense?)



→ ∃ localized states in bands

- yield long lived oscillations



Note: in general, these states are sensitive to disorder